

## EE 360 ELECTRIC Energy

### LECTURE 28

#### Power Transmission

The material covered in this lecture will be as follows:

⇒ To explain how electricity is transmitted from generation locations to load centers and points of utilization.

To explain how electricity is transmitted from generation locations to load centers and points of utilization.

⇒ To describe the details of overhead line components .

To describe the transmission components. ⇒

To describe the components of underground cables.

To describe the components of underground cables.

To list the factors those affect the choice of transmission voltage.

To list the factors those affect the choice of transmission voltage.

At the end of this lecture you should be able to:

⇒ Understand how lines are used to transmit power.

Understand how lines are used to transmit power.

⇒ understand the role of the components of transmission lines..

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#### 1. Introduction

Transmission lines carry power from generating plants to the distribution systems that feed electricity to domestic, commercial and industrial users.

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Electricity is normally generated away from load centers. This is because of environmental and safety reasons. Hydro resources may be at remote location

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Transmission lines can be overhead or underground cables.

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Electricity is usually sent over long distance through overhead power transmission lines.

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Underground power transmission is used only in densely populated areas (such as large cities) because of the high costs and losses.

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Most power is transmitted as 60 Hertz (cycles per second) alternating current (AC) power.

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AC power readily changed in voltage by transformers and is easily used in home appliances and motors. Some very specialized transmission uses direct current (DC) power.

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All references to transmission lines in this lecture and the subsequent lectures are in terms of three phase terms: line-line voltages etc.

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Transmission lines vary from a few kilometers long in an urban environment to over 1000 km for lines carrying power from remote hydroelectric plants.

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They may differ greatly in the amount of power carried. Transmission normally takes place at high voltage.

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The following list contains some general records and is only correct at the time of preparing his lecture.

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Highest transmission voltage (AC): 1150 kV (Kazakhstan)

Highest transmission voltage (DC): +/-600 kV (Brazil)

Longest transmission line: Democratic Republic of Congo (length: 1700km)

Longest submarine cables Baltic-Cable, Baltic Sea - (length of submarine/underground cable: 249 km, total length: 261 km) ..

The definition of the transmission voltage varies from a system to a system.

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In Saudi Arabia the highest transmission voltage is 380 kV.

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Other transmission voltages are also used :230 kV, 132 kV, 115-110 kV

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This is correct in 2006. Higher voltage may be realized in the near future.

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## 2. Transmission Line Construction

An overhead transmission line is made of conductor, insulators and a tower.

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An underground cable is made of conductor, insulation and is buried into ground.

An underground cable is made of conductor, insulation and is buried into ground.

The three phase conductors carry the electric current.

The three phase conductors carry the electric current.

Insulators provide support and electrically isolate the conductors.

Insulators provide support and electrically isolate the conductors.

Tower holds the insulators and conductors. It is firmly grounded with special foundation and

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Optional shield and ground conductors protect against lightning

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Figure 1 shows a typical transmission line tower together with conductors and insulators.

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Figure 1 Transmission Line

Figure 2 shows a typical transmission line tower.

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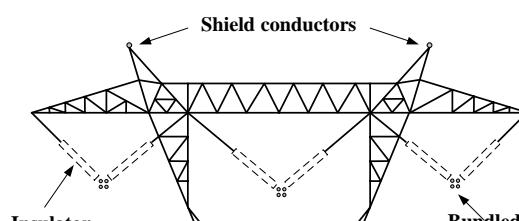


Figure 2 Transmission Tower

## 2. Overhead Line Components

### 2.1 Conductors

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Transmission line conductors are normally made from Aluminum with certain reinforcements.

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Copper is not usually at high voltage because of its costs even though it has a very low resistance.

Copper is not usually at high voltage because of its costs even though it has a very low resistance.

The conductors are made of aluminum strands which are reinforced by another material.

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Stranded conductors are simpler to manufacture, easier to handle and more flexible.

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The reinforcement, by steel for instance, provides a high strength-to-weight ratio.

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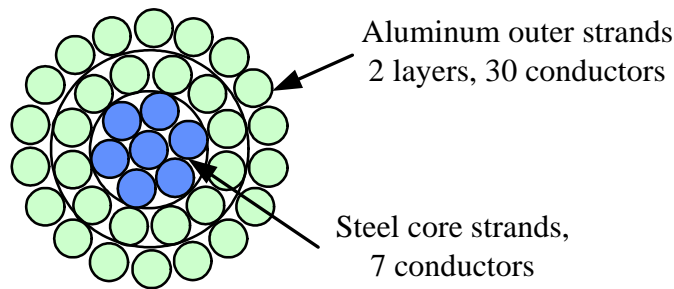
Aluminum conductors are classified as follows:

Aluminum conductors are classified as follows:

- Aluminum Conductor Steel Reinforced (ACSR)
- Aluminum Conductor Steel Reinforced (ACSR)
- All Aluminum Conductor (AAC); and
- All Aluminum Conductor (AAC); and
- 
- All Aluminum Alloy Conductor (AAAC)
- All Aluminum Alloy Conductor (AAAC)

Figure 3 shows a cross-section of an ACSR conductor.

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**Figure 3 An ACSR conductor**

The conductor is made of 30 Aluminum strands and 7 steel strands.  
**The conductor is made of 30 Aluminum strands and 7 steel strands.**

The electric current is carried by the aluminum strands while the steel strands provide mechanical support.

**The electric current is carried by the aluminum strands while the steel strands provide mechanical support**

Overhead line conductors are usually bare without any insulation.  
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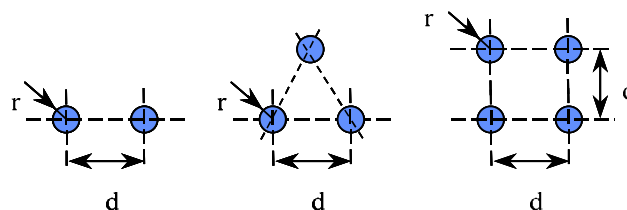
Bare conductors have excellent heat dissipation characteristics.  
**Bare conductors have excellent heat dissipation characteristics.**

Also insulating high voltage conductors will be economically prohibitive.  
**Also insulating high voltage conductors will be economically prohibitive.**

High voltage lines often may have more than one conductor per phase.  
**High voltage lines often may have more than one conductor per phase.**

This arrangement is referred to as Bundle-conductor arrangement.  
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Figure 4 shows arrangements for system of 2, 3 and 4 bundle conductors.  
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**Figure 4 Bundle Conductor arrangements**

Bundle conductors have lower electric strength at the surface. This controls the occurrence of corona.

**Bundle conductors have lower electric strength at the surface. This controls the occurrence of corona**

Corona is defined as the ionization of gas around transmission lines.  
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It is manifested by a hissing sound and in some extreme conditions a glowing light around the conductors.

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The subject of corona and its effects will be addressed in advanced EE courses.  
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## 2.2 Insulators

Insulators are used to support, anchor and insulate conductors from ground.  
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They are made of porcelain, glass and several synthetic materials.  
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Electrically, insulators must provide high resistance to leakage currents and they have to withstand certain voltage without damage.  
Electrically, insulators must provide high resistance to leakage currents and they have to withstand certain voltage without damage.

Mechanically, they must withstand the pull due to the conductor weight.  
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There are two types of insulators: pin and suspension types.  
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Suspension type are usually used for high voltage line. A number of insulators usually form a string between the conductors and the tower cross arms.  
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Figure 5 shows a typical insulator  
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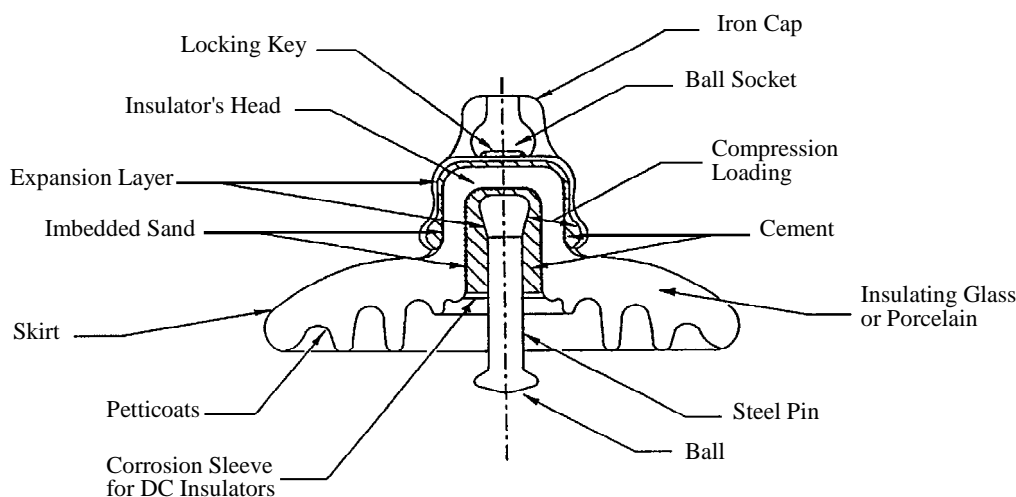


Figure 5 A Typical Transmission Insulator.

The number of insulators is dictated by the voltage level of the line.

The number of insulators is dictated by the voltage level of the line.  
Once again, the subject of insulators will be addressed in advanced EE courses.  
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### 2.3 Towers or Supporting structures

A structure is needed to keep conductors at a safe height from the ground.  
A structure is needed to keep conductors at a safe height from the ground.

It should also provides a acceptable distance between phase conductors to avoid arcing.  
It should also provides a acceptable distance between phase conductors to avoid arcing.

Wood and concrete poles are used for low voltage lines. High voltage lines use steel towers.  
Wood and concrete poles are used for low voltage lines. High voltage lines use steel towers.

Figure 2 earlier showed a transmission steel tower.  
Figure 2 earlier showed a transmission steel tower.

The design and height of the tower depends on many factors: transmission voltage, ground terrain, atmospheric conditions and environmental constraints  
The design and height of the tower depends on many factors: transmission voltage, ground terrain, atmospheric conditions and environmental constraints

### 3. Cable Components

The term cable refers to both underground and submarine cables.  
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Underground cables are buried below the ground while submarine cables are used for water crossings .

Underground cables are buried below the ground while submarine cables are used for water crossings.

Figure 6 shows a single core high voltage cable.

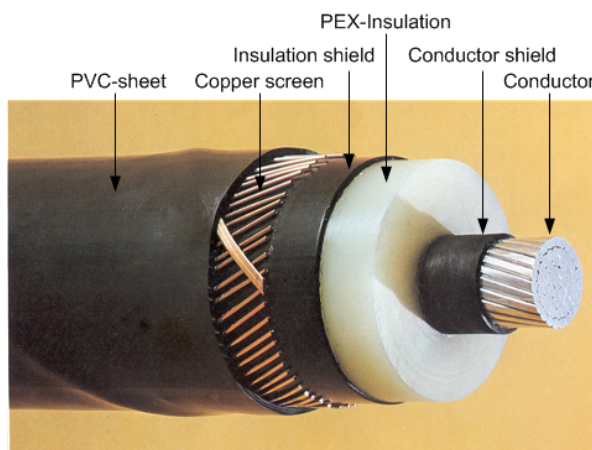


Figure 6 A Single Phase Cable

The cable components are:

1. Stranded phase Conductor.

**Stranded phase Conductor.**

2. A conductor sheath  
**A conductor sheath**
3. Insulating material: solid or oil-filled.  
**Insulating material: solid or oil-filled.**
4. Insulation shield .  
**Insulation shield .**
5. Copper screen or armor  
**Copper screen or armor**
6. External jacket ( mostly PVC).  
**External jacket ( mostly PVC).**

**Cable insulation material vary widely. The following are widely used**  
**Cable insulation material vary widely. The following are widely used.**

1. Oil impregnated paper
2. Gas such as SF<sub>6</sub>
3. Polyvinyl Chloride (PVC)
4. Cross linked Poly Ethylene (XLPE)

The choice of the insulation depends on the cable voltage.

**The choice of the insulation depends on the cable voltage.**

#### **4. Transmission Line Design**

Transmission line design is covered in details in advanced EE courses

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However, in this section, we will highlight the factors that influence the design.

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These are electrical, mechanical, environmental and economical factors.

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##### **a. electrical factors**

These dictate

1. Type, size, and number of conductors per phase

**Type, size, and number of conductors per phase**

2. The number of insulators per string and their arrangements.

**2. The number of insulators per string and their arrangements.**

3. Phase-to-phase clearance

**3. Phase-to-phase clearance**

4. Number of shield or ground conductors.

**4. Number of shield or ground conductors.**

##### **b. mechanical factors**

Conductors must be strong enough to support their weight in addition to wind .

**Conductors must be strong enough to support their weight in addition to wind .**

Towers must support conductors under vibration.

**Towers must support conductors under vibration.**

##### **c. Environmental**

- a. land usage and visual impact.

**c. Radio interference and corona noise**

- b. limits of electrostatic and electromagnetic fields
- b. limits of electrostatic and electromagnetic fields.
- c. Radio interference and corona noise.
- c. Radio interference and corona noise.

**d. Economical**

- 1. Capital costs
- 2. Maintenance and operation costs
- 1. Capital costs
- 2. Maintenance and operation costs

LECTURE 29

Transmission Line Parameters

The material covered in this lecture will be as follows:

- ⇒ To explain how to calculate line resistance.  
To explain how to calculate line resistance.
- ⇒ To calculate conductor Geometric Mean Distance (GMD) and Geometric Mean Radius (GMR) of conductors.  
To calculate conductor Geometric Mean Distance (GMD) and Geometric Mean Radius (GMR) of conductors.
- ⇒ How to calculate line inductance and capacitance .  
How to calculate line inductance and capacitance .

At the end of this lecture you should be able to:

- ⇒ Develop mathematical relationship to calculate line parameters.  
Develop mathematical relationship to calculate line parameters.
- ⇒ Test the relationship through a number of examples.  
Test the relationship through a number of examples.

1. Transmission Line Parameters

1.1 Line Resistance

Conductor resistance is function of :

Conductor resistance is function of :

The material resistivity,

The material resistivity,

Cross-sectional area

Cross-sectional area

Length of the conductor

Length of the conductor

Conductor operating temperature

Conductor operating temperature

Frequency of the current through conductor ( AC or DC)

Frequency of the current through conductor ( AC or DC)

Construction (stranding and spiraling)

Construction (stranding and spiraling)

The DC resistance R of conductor is given

The DC resistance R of conductor is given

$$R = \rho \frac{l}{A} \Omega \tag{1}$$

Where

$\rho$  = material resistivity in  $\Omega - m$

l = conductor length in meters

A = cross-sectional area in  $m^2$

Conductors resistance is affected by the operating temperature.

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Resistance increases with temperature. The rate of increase depends on the temperature coefficient of the material.

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The relationship to temperature is expressed as follows:

The relationship to temperature is expressed as follows:

$$R_2 = R_1[1 + \alpha(T_2 - T_1)] \quad (2)$$

Where

$R_1$  = resistance at temperature  $T_1$  in Ohms

$R_2$  = resistance at temperature  $T_2$  in Ohms

$T_1$  = reference temperature in  $^{\circ}\text{C}$ .

$T_2$  = operating temperature in  $^{\circ}\text{C}$ .

$\alpha$  = temperature coefficient

Several references and handbooks provide typical data on conductor resistivity and temperature coefficient as shown in Table 1.

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Table 1 Values of Conductor Resistivity and Temperature Coefficient [1]

Material	Resistivity $\rho$ in $\mu\Omega - cm$ at $20^{\circ}$	Temperature Coefficient $\alpha$ at $20^{\circ}$
<b>Aluminum</b>	<b>2.83</b>	0.0039
<b>Brass</b>	<b>6.4-8.4</b>	0.0020
Copper		
Hard drawn	1.77	0.00382
Annealed	1.72	0.00393
Iron	10.0	0.0050
Silver	1.59	0.0038
Steel	12-88	0.001-0.005

[1] "W.D. Stevenson, " Elements of Power System Analysis, 4<sup>th</sup> edition, McGraw-Hill, 1982

Resistance is also affected by the frequency of the operating current.

Resistance is also affected by the frequency of the operating current.

When AC current flows into the conductor, current is not uniformly distributed over the cross-sectional area.

When AC current flows into the conductor, current is not uniformly distributed over the cross-sectional area.

The current density is highest at the surface of the conductor.

The current density is highest at the surface of the conductor

The AC resistance is thus somewhat higher than DC resistance. This phenomena is called **Skin Effect**

The AC resistance is thus somewhat higher than DC resistance. This phenomenon is called **Skin Effect**.

A multiplying factor (1.2-1.3) is used to convert the DC resistance into 60 Hz resistance.

Moreover, transmission line conductors are constructed in a spiraled manner for better mechanical strength.

Moreover, transmission line conductors are constructed in a spiraled manner for better mechanical strength.

This in turn increases the length of the conductor. Thus a provision is needed to account for this manufacturing requirement.

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The conductor resistance is increased by few percentages to account for the spiraling effects.

The conductor resistance is increased by few percentages to account for the spiraling effects.

### Example 1

An solid aluminum conductor has a cross-sectional area of  $180 \text{ mm}^2$ . It is extended for 40 km. Find the conductor resistance at (a)  $20^\circ\text{C}$  (b)  $55^\circ\text{C}$ . The resistivity and temperature coefficient of Aluminum may be obtained from Table.

#### Solution

Using equation 1 and the data of Table 1

$$R = \rho \frac{l}{A} \Omega$$

Where

$$\rho = 2.83 \times 10^{-8} \Omega - m$$

$$l = 40,000 \text{ meters}$$

$$A = 180 \times 10^{-6} \text{ m}^2$$

$$\alpha \text{ at } 20^\circ = 0.0039$$

$$(a) R_{20} = (2.83 \times 10^{-8} \times 40,000) / (180 \times 10^{-6}) = 6.289 \Omega$$

(b) Using equation 2

$$R_{55} = R_{20} [1 + \alpha(T_2 - T_1)]$$

$$R_{55} = 6.289 [1 + 0.0039(55 - 20)] = 7.147 \Omega$$

## 4.2 Line Inductance

A current carrying conductor produces a magnetic field around the conductor.

A current carrying conductor produces a magnetic field around the conductor.

Magnetic field lines are concentric closed circles.

Magnetic field lines are concentric closed circles.

When the current changes, the magnetic flux changes and a voltage is induced.

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**Inductance** is a measure of the amount of magnetic flux produced for a given electric current.

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The SI unit of inductance is the henry (symbol: H). The symbol  $L$  is used for inductance, in honor of the physicist Heinrich Lenz.

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The inductance has the following relationship:

$$L = \frac{\Phi}{I} \quad \text{H} \quad (3)$$

Where

$L$  is the inductance in henries (H)

$I$  is the current in amperes,

$\phi$  is the magnetic flux in webers (Wb)

The quantity just defined is called *self-inductance*, because the magnetic field is created solely by the conductor that carries the current.

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When a conductor is coiled upon itself  $N$  number of times around the same axis (forming a solenoid), the current required to produce a given amount of flux is reduced by a factor of  $N$  compared to a single turn of wire.

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Thus, the inductance of a coil of wire of  $N$  turns is given by:

Thus, the inductance of a coil of wire of  $N$  turns is given by:

$$L = \frac{N \Phi}{I} = \frac{\lambda}{I} \quad \text{H} \quad (4)$$

Where  $\lambda$  is the total 'flux linkage'

The inductance of a single conductor is due to components of flux linkages.

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An internal flux linkage is given by

An internal flux linkage is given by

$$\lambda_{\text{int}} = \frac{\mu_0 I}{8\pi} \quad \text{Wb/m} \quad (5)$$

Where

$\mu_0$  = permeability of free space or air

$$= 4\pi \times 10^{-7} \text{ H/m}$$

The internal flux linkage is independent of the conductor dimensions.

The internal flux linkage is independent of the conductor dimensions.

The inductance due to internal flux is given by

The inductance due to internal flux is given by

$$L_{\text{int}} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \text{ H / m} \quad (6)$$

The external flux linkage is an expression of the magnetic flux outside the conductor surface into the space linking other metallic element

The external flux linkage is an expression of the magnetic flux outside the conductor surface into the space linking other metallic element

It depends on the conductor diameter and the distance into space being considered. It is give by

It depends on the conductor diameter and the distance into space being considered. It is give by

$$L_{\text{ext}} = \frac{\mu_0}{2\pi} \ln \frac{D}{r} = 2 \times 10^{-7} \ln \frac{D}{r} \text{ H / m} \quad (7)$$

Where

r= conductor radius in meters

D= distance between the conductor and another conductor in meters.

D= distance between the conductor and another conductor in meters.

The total inductance id given by

The total inductance id given by

$$L = 2 \times 10^{-7} \left( \frac{1}{4} + \ln \frac{D}{r} \right) \text{ H / m} \quad (8)$$

Equation 8 can be rewritten as:

Equation 8 can be rewritten as:

$$L = 2 \times 10^{-7} \left( \ln \frac{D}{r'} \right) \text{ H / m} \quad (9)$$

Where

$$r' = r e^{\frac{-1}{4}}$$

**It is referred as Geometric Mean Radius GMR**

It is normally written as  $D_s$  instead of  $r'$

It is normally written as  $D_s$  instead of  $r'$

Therefore equation 9 is written as

Therefore equation 9 is written as

$$L = 2 \times 10^{-7} \left( \ln \frac{D}{D_s} \right) \text{ H / m} \quad (10)$$

Equation 10 can be used represent of a single phase line where D represents the distance between the two conductors.

Equation 10 can be used represent of a single phase line where D represents the distance between the two conductors.

The inductance of three phase lines has to take into consideration the distances among the conductors and the fact that conductors are stranded.

The inductance of three phase lines has to take into consideration the distances among the conductors and the fact that conductors are stranded.

Moreover, in many high voltage lines have more than one conductor per phase (Bundles).  
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Also as per phase impedance is required to be symmetrical and equal; conductors are transposed along the line length.

Also as per phase impedance is required to be symmetrical and equal; conductors are transposed along the line length

This means that the interchanging the positions of the conductors every on third of the line length.

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Figure 6 shows a transposed line.

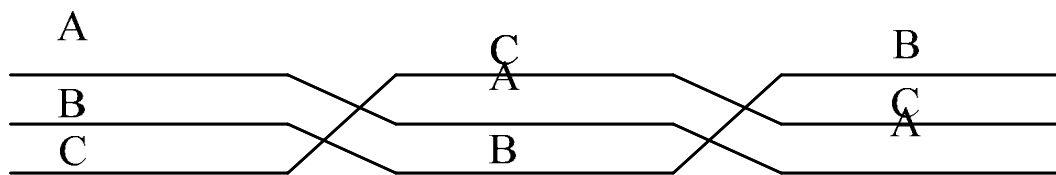


Figure 6 A transposed 3 phase transmission Line

Let us define a new term called Geometric Mean Distance (GMD) to reflect the line transposition  
 Let us define a new term called Geometric Mean Distance (GMD) to reflect the line transposition

$$GMD = \sqrt[3]{D_{AB} D_{AC} D_{BC}} \quad (11)$$

You may recall the conductors are not solid and are made of strands. To account for the stranding a  
 You may recall the conductors are not solid and are made of strands. To account for the stranding a

Geometric Mean Radius ( $D_s$ ) =  $n^2$  root of the product of  $n^2$  terms consisting of  $r'$  of every strands times the distance from each strand to all other strands within the group.

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Moreover for bundles conductors the following relationships apply

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Two-Bundle conductors separated by a distance  $d$ ,  $GMR_b = \sqrt{d * GMR_c}$

Two-Bundle conductors separated by a distance  $d$ ,  $GMR_b = \sqrt{d * GMR_c}$

Three-Bundle conductors separated by a distance  $d$ ,  $GMR_b = \sqrt[3]{d^2 GMR_c}$

Three-Bundle conductors separated by a distance  $d$ ,  $GMR_b = \sqrt[3]{d^2 GMR_c}$

Four-Bundle conductors separated by a distance  $d$ ,  $GMR_b = 1.09 \sqrt[4]{d^3 GMR_c}$

Four-Bundle conductors separated by a distance  $d$ ,  $GMR_b = 1.09 \sqrt[4]{d^3 GMR_c}$

Where

$GMR_b$  = GMR of the bundle.

$GMR_c$  = GMR of the individual conductor.

$D$  = distance between the bundle group.

In summary the inductance per phase of transposed 3 phase line with stranded conductors and bundled can be written as:

In summary the inductance per phase of transposed 3 phase line with stranded conductors and bundled can be written as:

$$L = 2 \times 10^{-7} \left( \ln \frac{GMD}{GMR} \right) H / m \quad (12)$$

### Example 2

The conductors of a three phase transmission line are arranged in a triangular configuration (delta). The distances between the conductors are as follows:

$$D_{AB} = 10 \text{ m}$$

$$D_{AC} = 6 \text{ m}$$

$$D_{BC} = 6 \text{ m}$$

The phase conductors are solid and have a radius of 0.4 cm.

Find the inductance per Km

### Solution

Use equation 11 to determine the GMD of the line

$$GMD = \sqrt[3]{10 \times 6 \times 6} = 7.113 \text{ m}$$

The GMR of the solid conductor is

$$r' = r e^{-\frac{1}{4}}$$

$$r' = 0.4 \times e^{-1/4} \times 10^{-2} = 0.311 \times 10^{-2}$$

### Using equation 12

### Using equation 12

$$L = 2 \times 10^{-7} \left( \ln \frac{GMD}{GMR} \right) = 2 \times 10^{-7} \ln(7.113 / (0.311 \times 10^{-2})) \quad H / m$$

$$L = 1.547 \times 10^{-6} \quad \text{H/m}$$

$$L = 1.547 \times 10^{-3} \text{H/km}$$

$$L = 1.547 \quad \text{mH/km}$$

## 4.3 Line Capacitance

**Capacitance** is a measure of the amount of electric charge stored (or separated) for a given electric potential.

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The capacitance is usually defined as the total electric charge placed on the object divided by the potential of the object:

The capacitance is usually defined as the total electric charge placed on the object divided by the potential of the object:

$$C = \frac{Q}{V} \quad \text{F} \quad (13)$$

Where

$C$  is the capacitance in farads, F  
 $Q$  is the charge in coulombs, C  
 $V$  is the potential in volts, V

**Capacitance** exists between any two conductors insulated from one another.

The conductors must have equal but opposite charge  $Q$ , and the voltage  $V$  is the potential difference between the two conductors.

The conductors must have equal but opposite charge  $Q$ , and the voltage  $V$  is the potential difference between the two conductors.

The SI unit of capacitance is the farad (F).

The SI unit of capacitance is the farad (F).

Transmission line conductors exhibit capacitance with respect to each other due to potential difference between them.

Transmission line conductors exhibit capacitance with respect to each other due to potential difference between them.

The amount of capacitance is a function of conductor size, spacing, and height above ground.

The amount of capacitance is a function of conductor size, spacing, and height above ground.

The charge on the conductor gives rise to an electric field with radial line into space.

The charge on the conductor gives rise to an electric field with radial line into space.

The intensity of the field is defined as force per unit charge (E). E is f

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E is function of the space permittivity and the distance from the charges conductor.

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$$E = \frac{Q}{2\pi\epsilon_0 x} \quad (14)$$

$\epsilon_0$  = permittivity of free space

$$= 8.85 \times 10^{-12} \text{ F/m}$$

$x$  = distance from conductor in m.

The potential difference is defined as the work done in moving a unit charge from on position  $D_1$  to position  $D_2$ .

The potential difference is defined as the work done in moving a unit charge from on position  $D_1$  to position  $D_2$ .

It is given by

$$V_{12} = \frac{Q}{2\pi\epsilon_0} \ln \frac{D_2}{D_1} \quad \text{V} \quad (15)$$

For a single phase line, conductors will have the same value of charge but of opposite polarity.

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Equation 14 can be reduced to

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$$V_{12} = \frac{Q}{\pi\epsilon_0} \ln \frac{D}{r} \quad \text{V} \quad (16)$$

Where

$D$  = separation between the two conductors

$r$  = radius of conductor.

The capacitance of the between conductors

The capacitance of the between conductors

$$C_{12} = \frac{\pi\epsilon_0}{\ln \frac{D}{r}} \quad \text{F/m} \quad (17)$$

The capacitance to ground then becomes

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$$C = \frac{2\pi\epsilon_0}{\ln \frac{D}{r}} \quad (18)$$

Using the same approach for the inductance of transposed 3 phase lines, the capacitance per phase

Using the same approach for the inductance of transposed 3 phase lines, the capacitance per phase

$$C = \frac{2\pi\epsilon_0}{\ln \frac{GMD}{r}} \quad \text{F/m} \quad (19)$$

Note that the actual conductor radius  $r$  is used rather than the GMR in case of the inductance.

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In case of bundled conductors, the following relationships are used

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Two-Bundle conductors separated by a distance  $d$ ,  $r_b = \sqrt{d * r}$

Two-Bundle conductors separated by a distance  $d$ ,  $r_b = \sqrt{d * r}$

Three-Bundle conductors separated by a distance  $d$ ,  $r_b = \sqrt[3]{d^2 r}$

Three-Bundle conductors separated by a distance  $d$ ,  $r_b = \sqrt[3]{d^2 r}$

Four-Bundle conductors separated by a distance  $d$ ,  $r_b = 1.09 \sqrt[4]{d^3 r}$

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In summary the capacitance per phase of transposed 3 phase line bundled conductors can be written as:

In summary the capacitance per phase of transposed 3 phase line bundled conductors can be written as:

$$C = \frac{2\pi\epsilon_0}{\ln \frac{GMD}{r_b}} \quad \text{F/m} \quad (20)$$

### Example 3

The conductors of a three phase transmission line are arranged in a triangular configuration (delta).

The distances between the conductors are as follows:

$$D_{AB} = 10 \text{ m}$$

$$D_{AC} = 6 \text{ m}$$

$$D_{BC} = 6 \text{ m}$$

The phase conductors are solid and have a radius of 0.4 cm.

Find the capacitance per Km

**Solution**

Using equation 19

$$C = \frac{2\pi\epsilon_0}{\ln \frac{GMD}{r}} \quad \text{F/m} \quad (19)$$

Use equation 11 to determine the GMD of the line

$$GMD = \sqrt[3]{10 \times 6 \times 6} = 7.113 \text{ m}$$

$$r = 0.4 \times 10^{-2}$$

$$C = \frac{2\pi\epsilon_0}{\ln \frac{GMD}{r}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln(7.113/0.4 \times 10^{-2})} = 7.43 \times 10^{-12} \text{ F/m}$$

$$C = 7.43 \times 10^{-9}$$

F/km

$$C = 7.43 \times 10^{-3}$$

$\mu$  F/km

## LECTURE 30

### Short and Medium Line Modeling

The material covered in this lecture will be as follows:

- ⇒ To develop model for transmission lines of various lengths.
- ⇒ To develop model for transmission lines of various lengths.
- ⇒ To calculate voltage and current in short and medium lines.
- ⇒ To calculate voltage and current in short and medium lines.

To understand the concepts of voltage regulation and efficiency in transmission lines.  
To understand the concepts of voltage regulation and efficiency in transmission lines..

At the end of this lecture you should be able to:

- ⇒ model transmission lines for calculating voltages and currents .
- ⇒ model transmission lines for calculating voltages and currents .
- ⇒ understand the power flow between two ends of the of a transmission line.
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#### 1. Introduction

The parameters of the transmission lines were determined in the previous lesson.

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The parameters are functions of conductor size and material, line configuration and others.

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They are expressed in per unit length of the line. Transmission lines vary in length.

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There are short lines of few kilometers and very long lines exceeding a thousand kilometer.

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A Transmission line is expected to deliver the power at the receiving end with minimum losses.

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Also the voltage at the receiving end should be as near as possible to the sending end voltage.

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The model used to determine the voltage and current in a line depends on the line parameters and its length.

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This and the following lessons deal with representation and performance under normal operating conditions.

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Transmission lines shall transmit power while satisfying the following requirement:

Transmission lines shall transmit power while satisfying the following requirement:

1. Voltage should remain as constant as possible through the entire line length.  
Voltage should remain as constant as possible through the entire line length
2. Lines losses should be as low as possible.  
Lines losses should be as low as possible.
3. Line conductors should not overheat due to the  $I^2R$  losses.  
Line conductors should not overheat due to the  $I^2R$  losses.

For the purpose of performance analysis, transmission lines are classified into short , medium and long lines according to their length.

For the purpose of performance analysis, transmission lines are classified into short , medium and long lines according to their length.

1. Short line whose length is less than 80 km.  
Short line whose length is less than 80 km.
2. Medium line whose length is between 80 km and 250 km.  
Medium line whose length is between 80 km and 250 km.
3. Long line whose length in more than 250 km.  
Long line whose length in more than 250 km.

The performance analysis vary from an approximate to detailed depending on the line length.  
The performance analysis vary from an approximate to detailed depending on the line length.

## 2. Definitions

Let us make the following definitions to be used throughout this lesson and subsequent lessons  
Let us make the following definitions to be used throughout this lesson and subsequent lessons

$r$  = line resistance per phase per unit length

$L$  = line inductance per unit length

$C$  = line capacitance per unit length

$V_R$  = phase voltage at the receiving end.

$I_R$  = receiving current

$V_S$  = phase voltage at the sending end

$I_S$  = sending end current

$S_R$  = three phase apparent power at receiving end

$S_S$  = three phase apparent power at sending end.

## 3. Short line Model and Representation

Capacitance of short lines is very small.

Capacitance of short lines is very small.

It is ignored in short lines without much error

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Figure 1 shows an equivalent circuit of a short line.

Figure 1 shows an equivalent circuit of a short line.

It is represented by a series impedance ,  $Z$ , which is given by

It is represented by a series impedance ,  $Z$ , which is given by

$$Z = (r + j \omega L)l \quad (1)$$

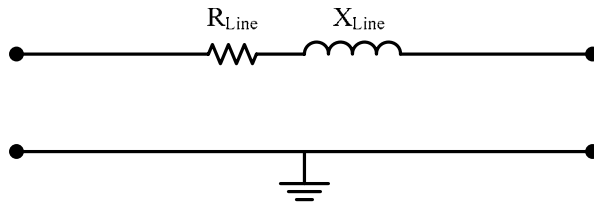
Where  $l$  = line length

$\omega$  = frequency

The impedance is written in terms of the total line resistance and line reactance as

The impedance is written in terms of the total line resistance and line reactance as

$$Z = (R + jX) \quad (2)$$



**Figure 1 Short line model**

Let us assume that the load at the receiving end has an apparent power of  $S_R$ .  
 Let us assume that the load at the receiving end has an apparent power of  $S_R$ .

The receiving end current is give by  
 The receiving end current is give by

$$I_R = \frac{S_R^*}{3 \cdot V_R} \quad (3)$$

The sending voltage will be equal to the receiving end voltage plus the series voltage drop due to the line impedance  
 The sending voltage will be equal to the receiving end voltage plus the series voltage drop due to the line impedance

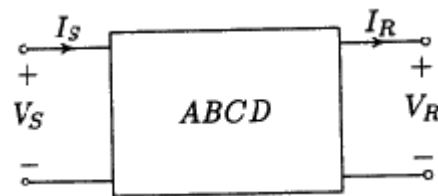
$$V_S = V_R + Z I_R \quad (4)$$

The sending current is equal to the receiving end current  
 The sending current is equal to the receiving end current

$$I_S = I_R \quad (5)$$

Equations 4 and 5 are the mathematical representation of a short line  
 Equations 4 and 5 are the mathematical representation of a short line

Transmission lines may be represented by a two port network as shown in figure 2.  
 Transmission lines may be represented by a two port network as shown in figure 2.



**Figure 2 Two Port network**

Equations 4 and 5 can be written as follows:  
 Equations 4 and 5 can be written as follows:

$$V_S = A V_R + B I_R \quad (6)$$

$$I_S = C V_R + D I_R \quad (7)$$

Where ABCD are constants called "Transmission Line constants"

Where ABCD are constants called "Transmission Line constants"

The ABCD constants are complex.

The ABCD constants are complex.

For a short line,

$$A = 1.0 \angle 0^\circ$$

$$B = Z$$

$$C = 0$$

$$D = A = 1.0 \angle 0^\circ$$

Once the sending end voltage is calculated, the voltage regulation of the line can be determined.

Once the sending end voltage is calculated, the voltage regulation of the line can be determined.

It is defined as

It is defined as

$$\text{Voltage Regulation (\%)} = \frac{V_{RO} - V_R}{V_R} \times 100 \quad (8)$$

Where  $V_{RO}$  = no load receiving end voltage

$V_R$  = full load voltage at the receiving end voltage.

At no load  $I_r = 0.0$ , equation 6 becomes

$$V_s = A V_R$$

The no load receiving end voltage is expressed in terms of the sending end voltage as

The no load receiving end voltage is expressed in terms of the sending end voltage as

$$V_{RO} = \frac{V_s}{A} \quad (9)$$

Finally, the transmission efficiency of the line is defined as ratio of the power received to the power sent.

Finally, the transmission efficiency of the line is defined as ratio of the power received to the power sent.

$$\eta = \frac{P_R}{P_S} \quad (10)$$

Where  $P_R$  = power at the receiving end

$P_S$  = power at the sending end

### Example 1

A 60 Hz, three phase 50 km line delivers 20 MW of power to load at 69 kV and a power factor of 0.8 lagging. The line has the following parameters:

$$r = 0.11 \Omega \text{ per km}$$

$$L = 1.11 \text{ mH per km}$$

C = negligible

Find

(i) the sending end voltage and current

(ii) voltage regulation

(iii) transmission efficiency

Solution

Let us first determine the line impedance using equation (1)

Let us first determine the line impedance using equation (1)

$$Z = (r + j \omega L)l$$

$$\omega = 2 \pi f = 2 \times 3.14159 \times 60 = 377$$

$$Z = (0.11 + j 377 \times 1.11 \times 10^{-3}) 50 = 5.5 + j 20.92 = 21.631 \angle 75.27 \text{ Ohms}$$

$$V_R = 69000 / \sqrt{3} = 39838.34 \text{ V}$$

$$I_R = \frac{20 \times 10^3}{\sqrt{3} \times 69 \times 0.8} = 209.19 \text{ A}$$

The current angle  $\theta_r = -36.87^\circ$

$$A = 1.0 \angle 0^\circ$$

$$B = 21.631 \angle 75.27 \text{ Ohms}$$

$$C = 0$$

$$D = A = 1.0 \angle 0^\circ$$

$$V_s = V_R + Z I_R = 39838.34 + (21.631 \angle 75.27^\circ \times 209.19 \angle -36.87^\circ)$$

$$39838.34 + 4525 \angle 38.4^\circ = 43384.55 + j 2810.69 = 43476 \angle 3.7^\circ$$

(i) the sending end line voltage = 75.3 kV

The sending end current is equal to the receiving end current = 209.19 A

(ii) The voltage regulation

**(ii) The voltage regulation**

$$\text{Voltage Regulation (\%)} = \frac{43476 - 39838}{39838} \times 100 = 9.13\%$$

(iii) The transmission efficiency

$$\text{The angle of the sending end current} = 3.7 + 36.87 = 40.57^\circ$$

Power factor at the sending end  $\text{pf}_s = 0.7596$  lagging

$$\text{The sending end power} = 3 \times V_s I_s \times \text{pf}_s = 3 \times 43476 \times 209.19 \times 0.7596 = 20.725 \text{ MW}$$

$$\text{The sending end power} = 3 \times V_s I_s \times \text{pf}_s = 3 \times 43476 \times 209.19 \times 0.7596 = 20.725 \text{ MW}$$

Therefore

$$\eta = \frac{P_R}{P_s} = \frac{20}{20.725} \times 100 = 96.50\%$$

#### 4. Medium Line Model and Representation

Capacitance of medium lines is significant and can not be ignored.

**Capacitance of medium lines is significant and can not be ignored.**

Also another element to be considered is the shunt conductance due to leakage current along insulators.

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This is referred to by the simple G . It is measured in Siemens.

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The total shunt admittance of a medium line is given by

**The total shunt admittance of a medium line is given by**

$$Y = (g + j\omega C)l \quad (11)$$

However, for the purpose of this lesson, the G term is neglected. Equation 11 becomes

$$Y = (j\omega C)l \quad (12)$$

Figure 3 shows an equivalent circuit representation of a medium line.

Figure 3 shows an equivalent circuit representation of a medium line.

In this representation, half of the shunt admittance (capacitance) is lumped at each end of the line.

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The model is referred to as  $\pi$  model for obvious reasons

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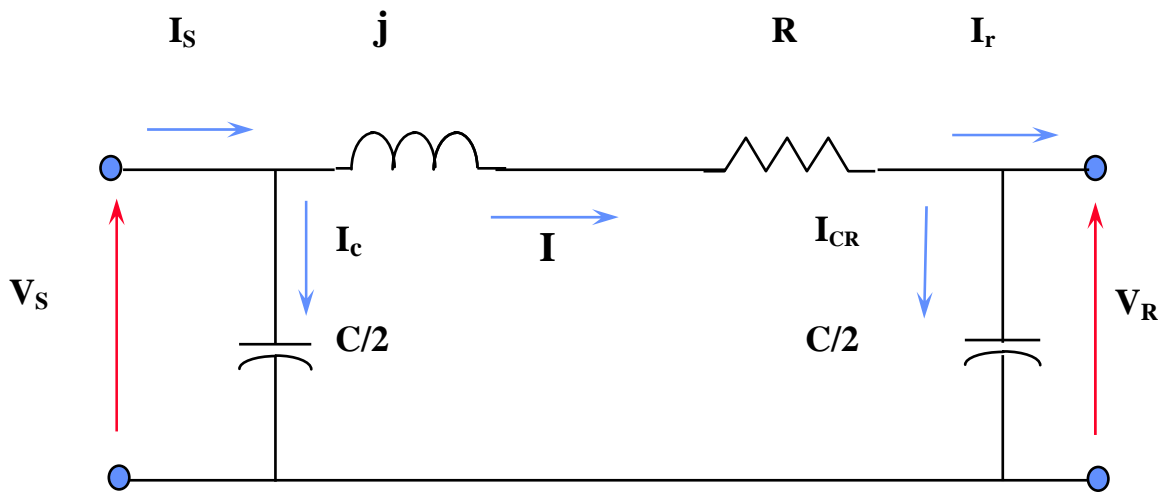


Figure 3 Nominal  $\pi$  Model of a medium line

Another model, which is not widely used, is shown in figure 4.

Another model, which is not widely used, is shown in figure 4.

The total shunt admittance (capacitance) is lumped at the center of the line and the total series impedance is divided into two equal parts.

The total shunt admittance (capacitance) is lumped at the center of the line and the total series impedance is divided into two equal parts.

The model is referred to as T model for obvious reasons.

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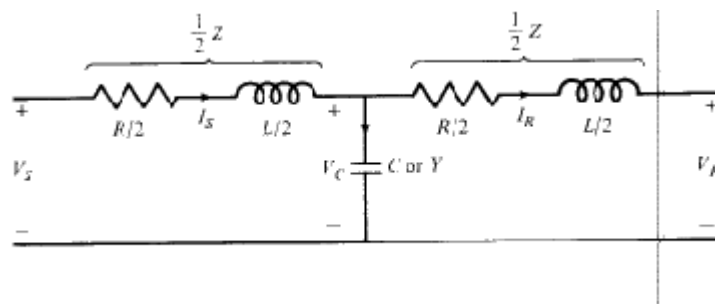


Figure 4 T-Model of a Transmission Line

The T-model will not be used in this lesson and all line performance calculations will use the  $\pi$  model of figure 3.

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Using KCL and KVL the following voltage-current relationships are derived.

Using KCL and KVL the following voltage-current relationships are derived.

$$I = I_R + I_{CR} \quad (13)$$

$$I = I_R + \frac{Y}{2}V_R \quad (14)$$

$$V_S = V_R + ZI \quad (15)$$

$$V_S = V_R \left(1 + \frac{ZY}{2}\right) + ZI_R \quad (16)$$

$$I_S = I + \frac{Y}{2}V_S \quad (17)$$

Substituting for I and V, equation 17 becomes

$$I_S = Y \left(I + \frac{ZY}{4}\right)V_R + \left(1 + \frac{ZY}{2}\right)I_R \quad (18)$$

Compare equations 6-7 with equations 16 & 18, the ABCD constants can be written  
Compare equations 6-7 with equations 16 & 18, the ABCD constants can be written

$$A = \left(1 + \frac{ZY}{2}\right) \quad (19)$$

$$B = Z \quad (20)$$

$$C = Y \left(1 + \frac{ZY}{4}\right) \quad (21)$$

$$D = \left(1 + \frac{ZY}{2}\right) \quad (22)$$

### Example 2

A 380 kV, 60 Hz, three phase 200 km delivers 400 MW of power at power factor of 0.8 lagging. The line has the following parameters:

$$r = 0.035 \Omega \text{ per km}$$

$$L = 0.9 \text{ mH per km}$$

$$C = 0.015 \mu\text{F per km}$$

Find

- (iv) the sending end voltage and current
- (v) voltage regulation
- (vi) transmission efficiency

### Solution

$$Z = (r + j\omega L)l$$

$$\omega = 2\pi f = 2\pi \times 3.14159 \times 60 = 377$$

$$Z = 7.0 + j67.86 \Omega$$

$$Y = j * 377 * 0.015 \times 10^{-6} * 200 = j0.0011$$

$$V_R = 380000 / \sqrt{3} = 219,399 \text{ V}$$

$$I_R = \frac{400 \times 10^3}{\sqrt{3} \times 380 \times 0.8} = 759.639 \text{ A}$$

The current angle  $\theta_r = -36.87^\circ$

$$A = 0.9616 + j0.0040$$

$$B = 7.0000 + j67.8600$$

$$C = -0.0000 + j0.0011$$

$$D = A$$

**(i) sending end voltage and current**

$$V_s = AV_R + BI_R$$

$$V_s = 2.4616 \times 10^5 + j3.8920 \times 10^4$$

$$|V_s| = 249.22 \text{ kV} \angle 8.98^\circ$$

The line voltage is

$$V_{sl} = 431.65 \text{ kV}$$

$$I_s = CV_R + DI_R$$

$$I_s = 5.8574 \times 10^2 - j1.9255 \times 10^2$$

$$I_s = 616.58 \angle -18.197^\circ$$

(ii) Voltage regulation

$$\text{Voltage Regulation (\%)} = \frac{249.22 - 219.399}{219.399} \times 100 = 13.59\%$$

Power factor angle at the sending end =  $8.98 + 18.197 = 27.177$

$$\text{Pf}_s = 0.889$$

The sending end power =  $3 \times V_s I_s \times \text{pf}_s = 3 \times 249220 \times 616.58 \times 0.889 = 409.8 \text{ MW}$

**The sending end power =  $3 \times V_s I_s \times \text{pf}_s = 3 \times 249220 \times 616.58 \times 0.889 = 409.8 \text{ MW}$**

$$\eta = \frac{P_R}{P_s} = \frac{400}{409.822} \times 100 = 97.60\%$$