

3.1 Three phase quantities

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360 Chapter 3 Three-phase Circuits

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Three Phase Circuit

Balanced three phase system

- Three generators produce voltages
 - same magnitude
 - 120 deg phase shift

Balanced load

$$\begin{aligned} i_a(t) &= I_M \cos(\omega t - \theta) \\ i_b(t) &= I_M \cos(\omega t - \theta - 120^\circ) \\ i_c(t) &= I_M \cos(\omega t - \theta - 240^\circ) \end{aligned}$$

$$\begin{aligned} v_{an}(t) &= V_M \cos(\omega t) \\ v_{bn}(t) &= V_M \cos(\omega t - 120^\circ) \\ v_{cn}(t) &= V_M \cos(\omega t - 240^\circ) = V_M \cos(\omega t + 120^\circ) \end{aligned}$$

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Three Phase Circuit

Balanced system

$$v_a(t) + v_b(t) + v_c(t) = 0 \quad i_a(t) + i_b(t) + i_c(t) = 0$$

Power

The instantaneous power is constant

$$p(t) = p_a(t) + p_b(t) + p_c(t) = 3 \frac{V_M I_M}{2} \cos(\theta) = 3 V_{rms} I_{rms} \cos(\theta)$$

Three phase power

$$\mathbf{S}_T = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 3 \mathbf{S}_1$$

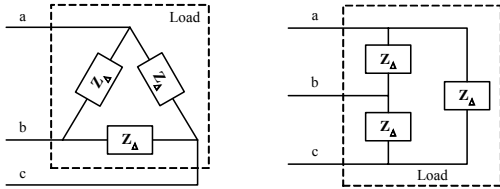
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Three Phase Circuit

Delta connection of load



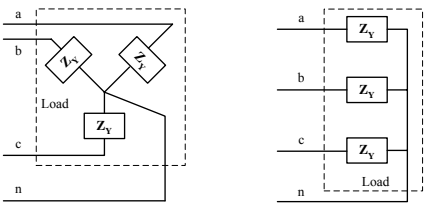
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Three Phase Circuit

Wye connection of Load



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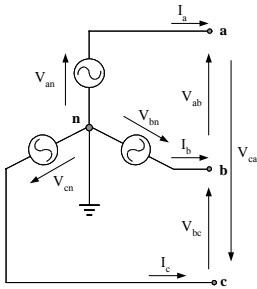
3.2 WYE conected generators

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Three Phase Circuit



Line-to-neutral voltages

$$\mathbf{V}_{an} = V_P \angle \delta$$

$$\mathbf{V}_{bn} = V_P \angle \delta - 120^\circ$$

$$\mathbf{V}_{cn} = V_P \angle \delta - 240^\circ$$

Balanced system

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

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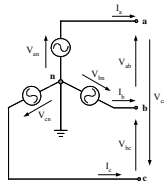
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Three Phase Circuit

$$\begin{aligned} \mathbf{V}_{ab} &= \mathbf{V}_{an} - \mathbf{V}_{bn} \\ &= \mathbf{V}_{an} - \mathbf{V}_{an} \angle -120^\circ \\ &= \mathbf{V}_{an} (1 - e^{-j120^\circ}) \\ &= \mathbf{V}_{an} \sqrt{3} \angle 30^\circ \end{aligned}$$

The two other can be calculated similarly

- The three line-to-line voltages, from the $n \rightarrow a \rightarrow b \rightarrow n$ loop



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Three Phase Circuit

The line to line voltages

$$\mathbf{V}_{ab} := \mathbf{V}_{an} - \mathbf{V}_{bn} \quad \mathbf{V}_{bc} := \mathbf{V}_{bn} - \mathbf{V}_{cn}$$

$$\mathbf{V}_{ca} := \mathbf{V}_{cn} - \mathbf{V}_{an}$$

The equations can be modified

$$\mathbf{V}_{ab} = \mathbf{V}_{an} \sqrt{3} \angle 30^\circ$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} \sqrt{3} \angle 30^\circ = \mathbf{V}_{an} \sqrt{3} \angle -90^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} \sqrt{3} \angle 30^\circ = \mathbf{V}_{an} \sqrt{3} \angle -210^\circ$$

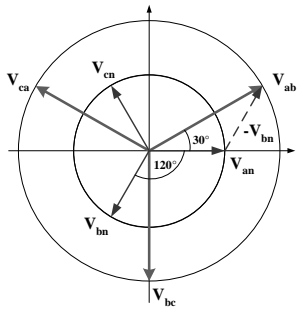
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Three Phase Circuit

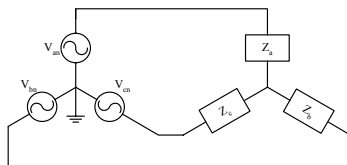
Vector diagram



3.3 WYE Connected Loads

Three Phase Circuit

Three wire system



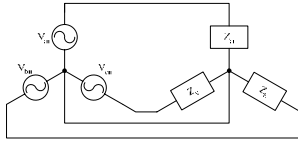
Three Phase Circuit

Four wire system

$$I_a = \frac{V_{an}}{Z_a} \quad I_b = \frac{V_{bn}}{Z_b} \quad I_c = \frac{V_{cn}}{Z_c}$$

$$S_T = S_a + S_b + S_c = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

$$I_0 = I_a + I_b + I_c \neq 0$$



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3.4 Delta Connected Loads

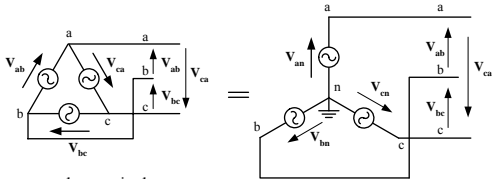
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Three Phase Circuit

Delta connected generators



Can be converted to equivalent wye

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} e^{-j30^\circ} \quad V_{bn} = \frac{V_{bc}}{\sqrt{3}} e^{-j30^\circ} \quad V_{cn} = \frac{V_{ca}}{\sqrt{3}} e^{-j30^\circ}$$

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Three Phase Circuit

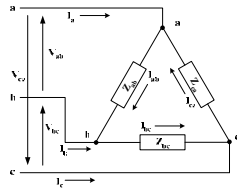
Delta connected load

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} \quad I_{bc} = \frac{V_{bc}}{Z_{bc}} \quad I_{ca} = \frac{V_{ca}}{Z_{ca}}$$

$$I_a = I_{ab} - I_{ca}$$

$$I_b = I_{bc} - I_{ab}$$

$$I_c = I_{ca} - I_{bc}$$



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3.6 Three phase power measurement

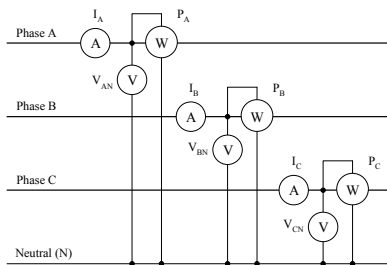
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Three Phase Circuit

Four wire system, Each phase measured separately



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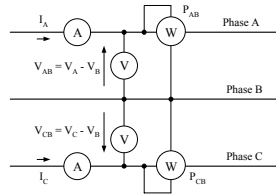
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Three Phase Circuit

Three wire system,

The three phase power is the sum of the two watt-meters reading

$$P_T = P_{AB} + P_{CB}$$



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Per Unit System

The voltage, current and impedance values are divided by base values and expressed in per unit or percentage values

Selection of base values: V_{base} (line-to-line) and S_{base} (magnitude of three phase complex power)

Calculate:

$$I_{base} = \frac{S_{base}}{\sqrt{3} V_{base}} \quad Z_{base} = \frac{V_{base}}{\sqrt{3} I_{base}} = \frac{V_{base}^2}{S_{base}}$$

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Per Unit System

Per Unit values using the base values are:

$$S_{pu} = \frac{|S|}{S_{base}} \quad I_{pu} = \frac{|I|}{I_{base}} \quad V_{pu} = \frac{|V|}{V_{base}} \quad Z_{pu} = \frac{|Z|}{Z_{base}}$$

Conversion from per unit value to ohm

$$Z_{pu} = \frac{|Z|}{Z_{base}} = \frac{S_{base}}{V_{base}^2} |Z| \quad |Z| = Z_{base} Z_{pu} = \frac{V_{base}^2}{S_{base}} Z_{pu}$$

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