EE 360 ELECTRIC Energy

LECTURE 15
DC Motors-I

The material covered in this lecture will be as follows:
The material covered in this lecture will be as follows:

1) The performance of DC motors
2) The torque speed relationship of DC motors

At the end of this lecture you should be able to:

To explain the performance of DC motors
To explain the performance of DC motors
Understand the torque speed relationship of DC motors
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1. Performance of DC Motors

The equivalent circuit of DC motors is similar to the equivalent circuit of DC generators, except that the direction of the current enters the armature circuit.

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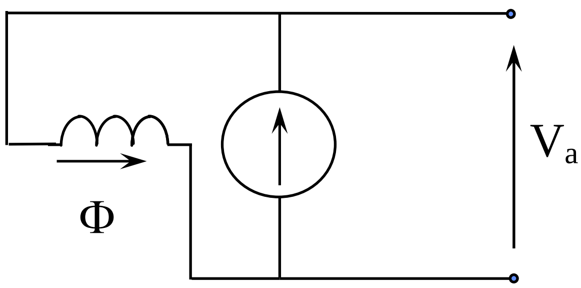

Figure 1 Shunt DC motor

Figure 1 shows the equivalent circuit of a shunt DC motor
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You may recall that the voltage generated in DC machine is function of both the speed and the flux. It is given by the following equation:.

You may recall that the voltage generated in DC machine is function of both the speed and the flux. It is given by the following equation:.

$$E_a = K_a \phi_m \omega \quad (1)$$

This is also expressed in terms of the field current as

$$E_a = K_a I_f \omega \quad (2)$$

The relationship between the generated and the terminal voltages is given by
The relationship between the generated and the terminal voltages is given by

$$V_a = E_a + I_a R_a \qquad (3)$$

Where
V_a= terminal voltage
I_a = armature current
R_a = armature resistance

The power developed by the machine is the product of the generated voltage and the armature current.

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$$P_{dev} = \omega T = E_a I_a \qquad (4)$$

The torque developed by the machine is given by

$$T = k_a \phi_m I_a \qquad (5)$$

Equation 4 shows that the machine torque is function of the magnetic flux and the current drawn by the machine.
Equation 4 shows that the machine torque is function of the magnetic flux and the current drawn by the machine.

The torque equation can also be written in terms of the field current and the machine constant.
The torque equation can also be written in terms of the field current and the machine constant.

$$T = k_a I_f I_a \qquad (6)$$

Using equations 1 and 3, the speed of a DC motor is expressed in equation 5.
Using equations 1 and 3, the speed of a DC motor is expressed in equation 5.

$$\omega = \frac{V_a - I_a R_a}{K_a \phi_m} \qquad (7)$$

Equation 5 is referred to as *speed equation* of DC motors. It contains all factors that affect the speed of the motor.
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1. 1 Speed Regulation

One measure of the performance of DC motors is the speed regulation. It is defined as
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It is defined as the change of speed from no load to full load operating condition of the motor.
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Equation 8 defines the speed regulation in terms of the no-load and full load speeds.
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$$\text{Speed Regulation} = \frac{w_{nl} - \omega_{fl}}{\omega_{fl}} x\, 100 \qquad (8)$$

Where

ω_{nl} **= no load speed of the motor in radians per second**

ω_{fl} **= full load speed of the motor in radians per second**

Equations 8 is also written as

Equations 8 is also written as

Speed Regulation= $\frac{n_{nl}-n_{fl}}{n_{fl}} x\,100$ **(9)**

n_{nl} **= no load speed of the motor in rpm**

n_{fl} **= full load speed of the motor in rpm**

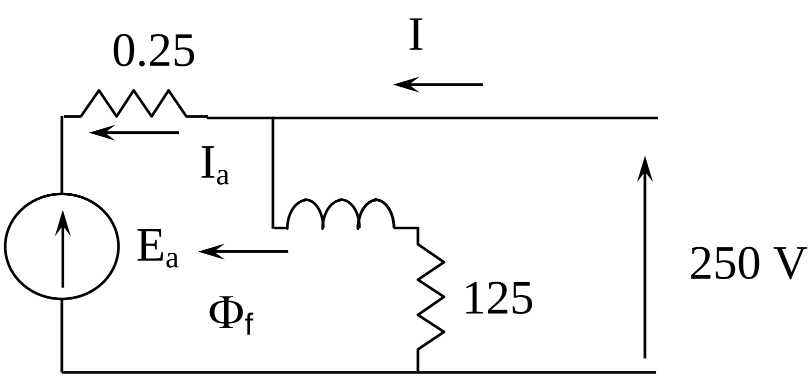
Example 1

A 25 kW, 250 V, shunt motor has an armature resistance of Ra = 0.25 ohm, and a field coil resistance of R_f = 125 ohms. At rated terminal voltage the motor current is 12 A and runs at 1000 rpm.

(a) Draw the equivalent circuit.
b) Calculate the motor constant
c) Calculate the speed, speed regulation and torque at full load conditions

Solution

(a) The Equivalent Circuit



(b) Machine constant:

Calculation of field current:

I_f = 250 / 125 = 2 A

The armature current at no load is:

I_a = 12 - 2 = 10 A

Generated voltage at no load is:

Ea = V_a - I_a Ra = 250 - (10)(0.25) = 247.5 V

No load speed:

$$\omega = \frac{2x\,\pi x\,1000}{60} = 104.72 \text{ rad/ sec}$$

- Machine constant:

$$E_a = K_a I_f \omega$$

$$247.5 = K_a x\, 2x\, 104.72$$

$$K_a = \frac{247.5}{2x\, 104.72} = 1.187$$

c. Speed and Torque at full load

Motor terminal input current:

I_t = 25000 / 250 = 100 A

- The armature current is:

  I_a = 100 - 2 = 98 A

- Generated voltage at full load is:

  $E_a = V_a - I_a Ra$ = 250 - (98)(0.25) = 225.5 V

- Motor speed at full load:

$$\omega = \frac{E_a}{K_a I_f}$$

$$\omega = \frac{225.5}{1.187x\, 2} = 94.98\, rad\ /\sec$$

The motor speed in rpm is

$$n = \frac{60x\, \omega}{2x\, \pi}$$

$$n = \frac{60x\, 94.98}{2x\, \pi} = 907.06\ rpm$$

Speed regulation= $\frac{1000 - 907.06}{907.06} x\, 100 \quad = 10.24\%$

- Torque at full load:

  Using equation 4, the motor torque is given as

$$T = \frac{E_a I_a}{\omega_m} = \frac{225.5x\, 98}{94.98} = 232.67\ Newtons - meter$$

Example 2

A DC motor develops a torque of 30 N-m. Determine the torque when the armature current is increased by 50% and the flux is reduced by 10%.

Solution

Recall that the motor torque is function of the armature current and flux as given by equation 5 earlier.

Recall that the motor torque is function of the armature current and flux as given by equation 5 earlier.

$$T = k_a \phi_m I_a$$

Let the initial toque and the corresponding variable be referred to by subscript (1)
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Let the final toque and the corresponding variable be referred to by subscript (2)
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$$T_1 = k_a \phi_{m1} I_{a1}$$
$$T_2 = k_a \phi_{m2} I_{a2}$$

$$\frac{T_2}{T_1} = \frac{k_a \phi_{m2} I_{a2}}{k_a \phi_{m1} I_{a1}}$$

$I_{a2}=1.5xI_{a1}$

$$\phi_{m2} = 0.9\phi_{m2}$$

$$\frac{T_2}{T_1} = \frac{k_a 0.9\phi_{m1} x 1.5 I_{a1}}{k_a \phi_{m1} I_{a1}}$$

$$T_2 = T_1 \frac{k_a 0.9\phi_{m1} x 1.5 I_{a1}}{k_a \phi_{m1} I_{a1}}$$

$$T_2 = 30x\,0.9x1.5 = 40.5 \;\; N - m$$

2. Speed Torque Characteristics of DC Motors

In order to understand the speed torque characteristics, we shall rewrite equations 5 and 7 again.
In order to understand the speed torque characteristics, we shall rewrite equations 5 and 7 again.

$$T = k_a \phi_m I_a \qquad (10)$$

$$\omega = \frac{V_a - I_a R_a}{K_a \phi_m} \qquad (11)$$

The armature current can be expressed in terms of the torque and the flux
The armature current can be expressed in terms of the torque and the flux

$$I_a = \frac{T}{k\,\phi_m} \qquad (12)$$

Substitute equation 12 into equation 11.
Substitute equation 12 into equation 11.

$$\omega = \frac{V_a}{K_a \phi_m} - \frac{R_a}{(K_a \phi_m)^2} T \qquad (13)$$

Equation 13 is referred to as the speed-torque equation.
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If the applied voltage and the flux remain constant, the speed will decrease linearly with the torque.
If the applied voltage and the flux remain constant, the speed will decrease linearly with the torque.
This is the case of an ideal shunt motor.
This is the case of an ideal shunt motor.
In an actual machine, the flux decreases slightly as the load increases due to armature reaction.
In an actual machine, the flux decreases slightly as the load increases due to armature reaction
This means there is a less reduction in speed.
This means there is a less reduction in speed.

In case of a series motor, the field is connected in series with the armature.
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The flux is thus a function of the armature current and the torque equation is written as follows.

The flux is thus a function of the armature current and the torque equation is written as follows.

$$T = k'_a I_a^2 \qquad (14)$$

The speed can than be expressed as

$$\omega = \frac{V_a}{K'_a I_a} - \frac{R_a}{K'} \qquad \mathbf{(15)}$$

Substitute for I_a in terms of the torque of equation 14.
Substitute for I_a in terms of the torque of equation 14.

$$\omega = \frac{V_a}{K''_a \sqrt{T}} - \frac{R_a}{K'} \qquad (16)$$

The speed of the series motor can be dangerously high at low or no-load torques.
The speed of the series motor can be dangerously high at low or no-load torques.

Therefore, series motors is never started with no-load connected to its shaft.
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The operation of a compound machine lies between the characteristics of shunt and series motors.
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When the two mmf are additive, the characteristics of the compound motor is more drooping than that of a shunt motor.
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When the two mmf are opposing, the characteristics of the compound motor lie above that of a shunt motor.

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Figure 3 shows typical speed-toque characteristics of DC motors.
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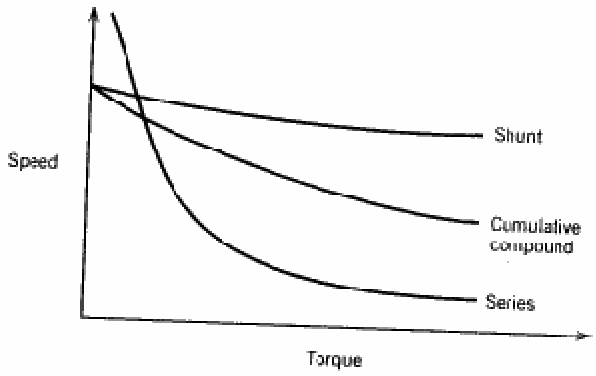

Figure 3 Speed-toque characteristics of DC motors.

Example 3

A 10-hp 230-V DC series motor has a line current of 37 A and a rated speed of 1200 rpm. The armature and series field resistances are 0.4 Ω and 0.2 Ω respectively. The total brush voltage drop is 2 V. Calculate the following:

a. Speed at a line current of 20 A.
b. No-load speed when the line current is 1 A.
c. Speed at 150% of the rated load when the line current is 60 A and the series field flux is 125% of the full load flux.

Solution

At full load I_{a1}, the generated back emf is given by
$E_{a1} = V_a - I_{a1}(R_s + R_a)$-Brush drop

$E_{a1} = 230-37(0.4+0.2)-2 = 205.8$ V

(a) When the load current I_{a2} is 20 A,
$E_{a2} = 230-20(0.4+0.2)-2 = 216$ V

$$\frac{E_{a2}}{E_{a1}} = \frac{K_a \phi_2 \omega_2}{K_a \phi_1 \omega_1} = \frac{K'_a I_{a2} n_2}{K'_a I_{a1} n_1}$$

The speed n_2 is given by

$$n_2 = \frac{E_{a2} I_{a1}}{E_{a1} I_{a2}} n_1 = \frac{(216) x\, 37}{(205.8) x\, 20} x\, 1200 = 2330\, rpm$$

(b) No-load speed
The armature current $I_{a3} = 1.0$ A

E_{a3}= 230-1(0.4+0.2)-2= 227.4 V

$$n_3 = \frac{E_{a3} I_{a1}}{E_{a1} I_{a3}} n_1 = \frac{(227.4) x\, 37}{(205.8) x\, 1} x\, 1200 = 49{,}060\, rpm$$

This confirms the statement that DC series motors should never be run at no-load as the speed is dangerously high.

c. Speed at 60 A armature current

I_{a4}= 60.0 A

E_{a4}= 230-60(0.4+0.2)-2= 192 V

The series flux is 125% of the full load flux.

$$\phi_4 = 1.25\phi_1$$

$$n_4 = \frac{E_{a4}\phi_1}{E_{a1}\phi_4} n_1 = \frac{(192) x\, \phi_1}{(205.8) x\, 1.25\phi_1} x\, 1200 = 1399 rpm$$