# EE 360 ELECTRIC Energy

**LECTURE 02**
**Power in Three Phase Circuits**

The material covered in this lecture will be as follows:

- ⇒ To understand the concepts of power in three phase circuits.
  To understand the concepts of power in three phase circuits
- ⇒ To know how to calculate power in Wye-connected circuits
  To know how to calculate power in Wye-connected circuits.

.

- ⇒ To know how to calculate power in Delta-connected circuits.
  To know how to calculate power in Delta-connected circuits

At the end of this lecture you should be able to:

- ⇒ Understand the definition of three phase power.
  Understand the definition of three phase power.
- ⇒ Find the components of the three phase power in a Y-connected system.
  Find the components of the three phase power in a Y-connected system.
- ⇒ Find the components of the three phase power in a Delta-connected system.

  Find the components of the three phase power in a Delta-connected system.

**1. Definition of Power in Single Phase circuits.**

In order to understand the concepts of power in three phase circuits, students are advised to review the material of courses EE 201 and EE 205.

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We can however summarize the power definitions is a single phase circuit
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**Real Power**

$$P = V_{rms} * I_{rms} * \cos(\theta) \qquad (1)$$

**Units are Watt**

**Real Power**

$$P = V_{rms} * I_{rms} * \cos(\theta) \qquad (1)$$

**Units are Watt**

**Reactive Power**

$$Q = V_{rms} * I_{rms} * \sin(\theta) \qquad (2)$$

$$Q = V_{rms} * I_{rms} * \sin(\theta) \qquad (2)$$

**The apparent power**

$$S = |V_{rms}| * |I_{rms}| = \sqrt{P^2 + Q^2} \qquad (3)$$

**The apparent power**

$$S = |V_{rms}| * |I_{rms}| = \sqrt{P^2 + Q^2}$$

**For convenience of writing, the absolute simple will be dropped. Equation 3 is the written as**

**For convenience of writing, the absolute simple will be dropped. Equation 16 is the written as**

$$S = V_{rms} * I_{rms} \qquad \textbf{(4)}$$

**Power Factor**

$$pf = \cos(\theta) \qquad \textbf{(5)}$$

**Power Factor**

$$pf = \cos(\theta)$$

## 2. Power in Balanced Three Phase circuits

Let us define the following variables
$S_p$= apparent power per phase
$S_T$= total apparent power

$P_p$= real power per phase
$Q_p$= reactive power per phase
$P_T$ = Total real power
$Q_T$= Total reactive power

Let us define the following variables
$S_p$= apparent power per phase
$S_T$= total apparent power

$P_p$= real power per phase
$Q_p$= reactive power per phase
$P_T$ = Total real power
$Q_T$= Total reactive power

In a balanced three phase system, let the phase voltage given by equations (6-8)
In a balanced three phase system, let the phase voltage given by equations (19-21)

$$v_a(t) = V_m \cos(\omega t + \alpha) \qquad (6)$$

$$v_b(t) = V_m \cos(\omega t + \alpha - 120) \qquad (7)$$

$$v_c(t) = V_m \cos(\omega t + \alpha - 240) \qquad (8)$$

$$v_a(t) = V_m \cos(\omega t + \alpha) \qquad (6)$$

$$v_b(t) = V_m \cos(\omega t + \alpha - 120) \qquad (7)$$

$$v_c(t) = V_m \cos(\omega t + \alpha - 240) \qquad (8)$$

Let the phase currents be given by equations 9-11

Let the phase currents be given by equations 9-11

$$i_a(t) = I_m \cos(\omega t + \beta) \qquad \textbf{(9)}$$

$$i_b(t) = I_m \cos(\omega t + \beta - 120) \quad \textbf{(10)}$$

$$i_c(t) = I_m \cos(\omega t + \beta - 240) \quad \textbf{(11)}$$

$$i_a(t) = I_m \cos(\omega t + \beta) \quad \textbf{(9)}$$

$$i_b(t) = I_m \cos(\omega t + \beta - 120) \quad \textbf{(10)}$$

$$i_c(t) = I_m \cos(\omega t + \beta - 240) \quad \textbf{(11)}$$

The instantaneous power in three phase circuit is as follows:
The instantaneous power in three phase circuit is as follows:

$$p(t) = V_m * I_m (\cos(\omega t + \alpha)\cos(\omega t + \beta) + (\cos(\omega t + \alpha - 120^0)\cos(\omega t + \beta - 120^0) + (\cos(\omega t + \alpha - 240^0)\cos(\omega t + \beta - 240^0)) \quad \textbf{(12)}$$

**Applying trigonometric identity**
**Applying trigonometric identity**

$$\cos(A) * \cos(B) = \frac{1}{2}(\cos(A - B) + \cos(A + B))$$

$$p(t) = \frac{V_m * I_m}{2}(3 * \cos(\alpha - \beta) + (\cos(2\omega t + \alpha - \beta) + \cos(2\omega t + \alpha - \beta - 120^0) + (\cos(2\omega t + \alpha - \beta - 240^0)) \quad \textbf{(13)}$$

**Let** $\phi = 2\omega t + \alpha - \beta$
**Let** $\phi = 2\omega t + \alpha - \beta$

**Equation 25 becomes**
**Equation 25 becomes**

$$p(t) = \frac{V_m * I_m}{2}(3 * \cos(\alpha - \beta) + (\cos(\phi) + \cos(\phi - 120^0) + (\cos(\phi - 240^0)) \quad \textbf{(14)}$$

**The terms involving** $\phi$ **will add up to zero. Equation 14 then becomes**
**The terms involving** $\phi$ **will add up to zero. Equation 14 then becomes**

$$p(t) = \frac{V_m * I_m}{2}(3 * \cos(\alpha - \beta)) \quad \textbf{(15)}$$

**Equation 28 can be rewritten in terms of the rms values of the voltage and current.**
**Equation 28 can be rewritten in terms of the rms values of the voltage and current.**

$$p(t) = 3 * V_{rms} * I_{rms} \cos(\alpha - \beta) \quad \textbf{(16)}$$

**The total instantaneous power is independent of time.** This is a major advantage of the three phase system.
**The total instantaneous power is independent of time.** This is a major advantage of the three phase system.

Three phase equipment will not be subjected to time-varying power.

Three phase equipment will not be subjected to time-varying power.

**The average power is therefore written as**
**The average power is therefore written as**

$$P_T = 3 * V_{rms} * I_{rms} \cos(\theta) \quad \textbf{(17)}$$

Where $\theta$ is defined the angle between the phase voltage and current. This equal to $(\alpha - \beta)$
Where $\theta$ is defined the angle between the phase voltage and current. This equal to $(\alpha - \beta)$

Similarly, the total reactive and apparent powers are given in equations 18 & 19 respectively.
Similarly, the total reactive and apparent powers are given in equations 31 & 32 respectively.

$$Q_T = 3 * V_{rms} * I_{rms} \sin(\theta) \quad \textbf{(18)}$$

$$S_T = 3 * V_{rms} * I_{rms} \quad \textbf{(19)}$$

$$Q_T = 3 * V_{rms} * I_{rms} \sin(\theta) \quad \textbf{(18)}$$

$$S_T = 3 * V_{rms} * I_{rms} \quad \textbf{(19)}$$

**Example 1**

A 3-phase balanced load has a phase voltage 120-V . The load has an impedance per phase equals to $12\angle 30^0 \Omega$ . Find
(i) The phase current
(ii) The real power per phase
(iii) Total real power
(iv) Total reactive power
(v) Total apparent power

**Solution**

**(i)**

$$\mathbf{I_a} = \frac{Phase\,voltage}{impedance\ per\ phase} = \frac{\mathbf{V}_{rms}}{\mathbf{Z_a}}$$

$$I_a = 120\angle 0^0 / 12\angle 30^0$$

$$I_a = 10\angle -30^0\, A.$$

(ii) Real Power per phase

$$P = V_{rms} * I_{rms} * \cos(\theta) = 120 * 10 * \cos(30)$$

$$P = 1039.23\, W$$

**(iii) Total real power**

$$P_T = 3 * V_{rms} * I_{rms} \cos(\theta) = 3 * 120 * 10 * \cos(30) = 3117.69 W$$

**(iv) Total reactive power**

$$Q_T = 3 * V_{rms} * I_{rms} \sin(\theta) = 3 * 120 * 10 * \sin(30) = 1800 Var$$

**(v) Total Apparent power**

$$S_T = 3 * V_{rms} * I_{rms} = 3 * 120 * 10 = 3600 VA$$

## 2.1 Power in a Balanced Y-connected circuits

Power in a Balanced Y-connected circuits

Figure 1 shows a Y-connected source supplying a Y-connected load.
Figure 1 shows a Y-connected source supplying a Y-connected load.

The objective is to determine the power in a Y-connected load
The objective is to determine the power in a Y-connected load

We start by writing the power expression for phase a using equation (1)

$$P = V_{rms} * I_{rms} * \cos(\theta)$$

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$$P = V_{rms} * I_{rms} * \cos(\theta)$$

The load voltage is equal to the supply phase voltage $V_{an}$ .
The load voltage is equal to the supply phase voltage $V_{an}$ .

This is because there is no voltage drop between the source and the load.
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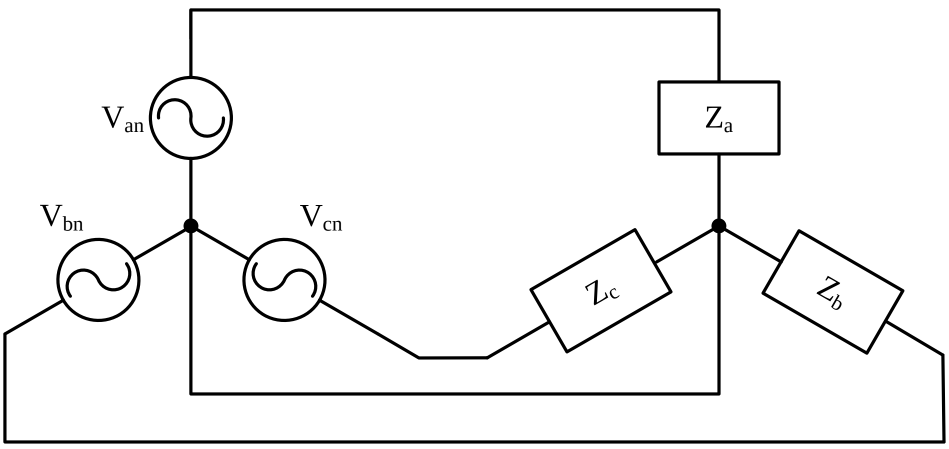


**Figure 1**

The current in phase a is given by
The current in phase a is given by

$$\mathbf{I_a} = \frac{\mathbf{V_{an}}}{\mathbf{Z_a}} \qquad (20)$$

The current will have a magnitude and angle depending on the value and type of the load impedance.
The current will have a magnitude and angle depending on the value and type of the load impedance.

$$\mathbf{I_a} = |I_a| \angle -\theta \quad (21)$$

The power absorbed by phase a is

The power absorbed by phase a is

$$P_a = V_{an} * I_a * \cos(\theta_a) \quad (22)$$

Where

$V_{an}$ = the magnitude of the voltage per phase for phase a.

$I_a$ =the magnitude of the current in phase a

$\theta_a$ = phase angle of phase a

$V_{an}$ = the magnitude of the voltage per phase for phase a.

$I_a$ =the magnitude of the current in phase a

$\theta_a$ = phase angle of phase a

Similar expressions can be written for the power associated with phase b & c.

Similar expressions can be written for the power associated with phase b & c.

$$P_b = V_{bn} * I_b * \cos(\theta_b) \quad (23)$$

$$P_c = V_{cn} * I_c * \cos(\theta_c) \quad (24)$$

For a balanced system

For a balanced system

$$V_{an} = V_{bn} = V_{cn}$$

$$I_a = I_b = I_c$$

$$\theta_a = \theta_b = \theta_c$$

Therefore

$P_a = P_b = P_c$

The general power equation can then be written as

The general power equation can then be written as

$$P_p = V_p * I_p * \cos(\theta) \qquad (25)$$

Where

$P_p$ = The average power per phase

$V_p$ = the magnitude of the phase voltage

$I_p$ =the magnitude of the phase current

$\theta$ = the angle between the phase current and voltage

The total power of the Y-connected load is give by equation (26).

The total power of the Y-connected load is give by equation (26).

$$P_T = 3 * V_p * I_p * \cos(\theta) \qquad (26)$$

In Y-connected system the relationship between the line and phase quantities are give by (27-28)

In Y-connected system the relationship between the line and phase quantities are give by (27-28)

$$V_l = \sqrt{3} * V_p \qquad (27)$$

$$I_l = I_p \qquad (28)$$

$V_l$ = the magnitude of the line voltage

$I_l$ =the magnitude of the line current

Using equation (40), substitute for $V_p$ & $I_p$ in terms of $V_l$ & $I_l$ as given in 27-28.

Using equation (40), substitute for $V_p$ & $I_p$ in terms of $V_l$ & $I_l$ as given in 27-28.

The total average power is written as

The total average power is written as

$$P_T = \sqrt{3} * V_l * I_l * \cos(\theta) \qquad (29)$$

Similar expressions can be written for the reactive power, apparent power and complex power

Similar expressions can be written for the reactive power, apparent power and complex power

$$Q_T = \sqrt{3} * V_l * I_l * \sin(\theta) \tag{30}$$

$$S_T = \sqrt{3} * V_l * I_l \tag{31}$$

$$\overline{S} = \sqrt{3} * V_l * I_l^* \angle\theta \tag{32}$$

**Example 2**

A 4-wire Y-connected 3-phase has a line voltage of 173.2 V is connected to Y-connected balanced load. The impedance per phase is given as 5+j5 Ohms. Find
(i) the total three phase power delivered to the load.
(ii) the total reactive power of the load
(iii) the total apparent power of the load. load.

Solution

Using Phase a as a reference

$V_{ab} = 173.2\angle 0^0$

$V_{bc} = 173.2\angle -120^0$

$V_{ca} = 173.2\angle -240^0$

The line voltage $V_l = 173.2V$

The phase voltage ate the load are given as follows:

$V_{an} = 100\angle 0 - 30^0$

$V_{bn} = 100\angle -150^0$

$V_{cn} = 100\angle -270^0$

The line-line voltages at the load are given by:

(ii) In Y-connected load , the phase and line currents are equal.

The load impedance per phase

$Z_a = 5 + j5 = 5\sqrt{2}\angle 45^0$

The phase current is calculated as follows

$$\mathbf{I}_p = \frac{\mathbf{V_{an}}}{\mathbf{Z_a}}$$

$$I_p = \frac{100\angle -30^0}{5\sqrt{2}\angle 45}$$

$$I_p = 14.14\angle -75^0 A.$$

$$I_l = I_p = 14.14A$$

$\theta$ = the angle between the phase current and voltage=$45^0$

(i) Total 3-phase power, using equation 29:

$$P_T = \sqrt{3} * V_l * I_l * \cos(\theta) = \sqrt{3} * 173.2 * 14.14 * \cos(45) = 3000W$$

(ii) Total Reactive power using equation 30:

$$Q_T = \sqrt{3} * V_l * I_l * \sin(\theta) = \sqrt{3} * 173.2 * 14.14 * \sin(45) = 3000Vars$$

(iii) Total Apparent power using equation 31:

$$S_T = \sqrt{3} * V_l * I_l = \sqrt{3} * 173.2 * 14.14 = 4242\ VA$$

## 2.2 Power in a Balanced Delta ( or $\Delta$) -connected circuits

**Power in a Balanced Delta ( or $\Delta$) -connected circuits**

The objective is to determine the power in a $\Delta$-connected load

The objective is to determine the power in a $\Delta$-connected load

The calculation for power in $\Delta$-connected load will follow the same procedure for the Y-connected.

The calculation for power in $\Delta$-connected load will follow the same procedure for the Y-connected.

Figure 2 shows a 3-phase source supplying a $\Delta$-connected load.

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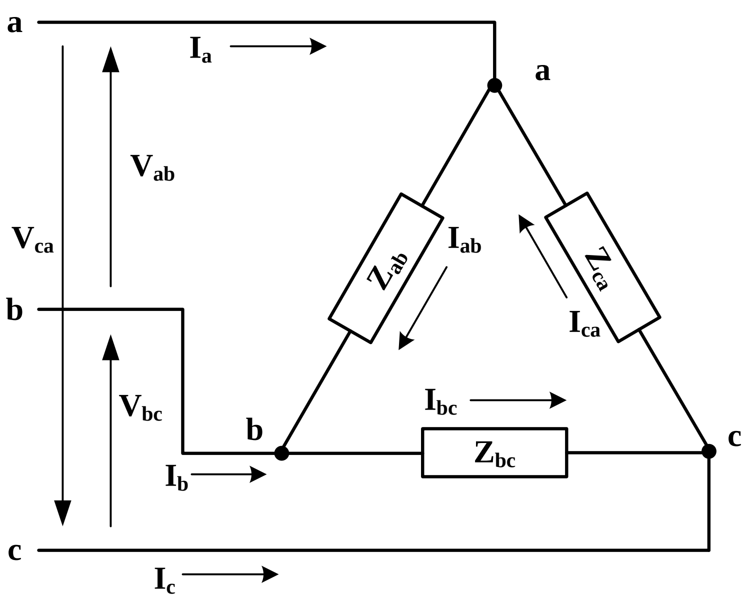


Figure 2

The power absorbed in phase a is given

The power absorbed in phase a is given

$$P_a = V_{ab} * I_{ab} * \cos(\theta_a) \quad (33)$$

Where

$V_{ab}$ = the magnitude of the voltage per phase for phase a.

$I_{ab}$ =the magnitude of the phase current in phase a

$\theta_a$ = phase angle of phase a

Similar expressions can be written for the power associated with phase b & c.

Similar expressions can be written for the power associated with phase b & c.

$$P_b = V_{bc} * I_{bc} * \cos(\theta_b) \quad (34)$$

$$P_c = V_{ca} * I_{ca} * \cos(\theta_c) \quad (35)$$

For a balanced system

For a balanced system

$V_{ab} = V_{bc} = V_{ca}$

$I_{ab} = I_{bc} = I_{ca}$

$\theta_a = \theta_b = \theta_c$

Therefore

$P_a=P_b=P_c$

The general power equation can then be written as

The general power equation can then be written as

$$P_p = V_p * I_p * \cos(\theta_p) \quad (36)$$

Where

$P_p$ = The average power per phase

$V_p$ = the magnitude of the phase voltage

$I_p$ =the magnitude of the phase current

$\theta$ = phase angle of the current

The total power of the $\Delta$-connected load is give by equation (37).

The total power of the $\Delta$-connected load is give by equation (37).

$$P_T = 3 * V_p * I_p * \cos(\theta) \qquad (37)$$

In $\Delta$-connected system the relationship between the line and phase quantities are give by (38-39)

In $\Delta$-connected system the relationship between the line and phase quantities are give by (38-39)

$$V_l = V_p \qquad (38)$$

$$I_l = \sqrt{3} * I_p \qquad (39)$$

$V_l$ = the magnitude of the line voltage

$I_l$ =the magnitude of the line current

Using equation (38), substitute for $V_p$ & $I_p$ in terms of $V_l$ & $I_l$ as given in 38-39.

Using equation (39), substitute for $V_p$ & $I_p$ in terms of $V_l$ & $I_l$ as given in 38-39.

The total average power is written as

The total average power is written as

$$P_T = \sqrt{3} * V_l * I_l * \cos(\theta) \qquad (40)$$

Similar expressions can be written for the reactive power, apparent power and complex power.

Similar expressions can be written for the reactive power, apparent power and complex power.

$$Q_T = \sqrt{3} * V_l * I_l * \sin(\theta) \qquad (41)$$

$$S_T = \sqrt{3} * V_l * I_l \qquad (42)$$

$$\overline{S} = \sqrt{3} * V_l * I_l^* \angle\theta \qquad (43)$$

Note
The equations for the power expressions in Y-connected load (29-32) and the $\Delta$-connected load ( 40-43) are identical.
The equations for the power expressions in Y-connected load (29-32) and the $\Delta$-connected load ( 40-43) are identical.

But power values are not equal for the same load when connected in Y or $\Delta$ connection.
But power values are not equal for the same load when connected in Y or $\Delta$ connection.

**Example 3**

A Y-connected 3-phase source has a line voltage of 173.2 V is connected to $\Delta$ -connected balanced load. The impedance per phase is given as 5+j5 Ohms. Find
(i) the total three phase power delivered to the load.
(ii) the total reactive power of the load
(iii) the total apparent power of the load. load.

**Solution**

For a $\Delta$ connected load, the phase and line voltages are equal.
For a $\Delta$ connected load, the phase and line voltages are equal.

The voltage between the two lines( or phases) is $173.2\ \angle 0^0$ V.

$V_{ab}$=$V_L$= 173.2-V.
The load impedance per phase
$Z_{ab} = 5 + j5 = 5\sqrt{2}\angle 45^0$

(i) The phase current $I_{ab} = \dfrac{V_{ab}}{Z_{ab}}$

The phase current $I_{ab} = \dfrac{173.2\angle 0}{5\sqrt{2}\angle 45^0} = 24.5\angle -45A$

The line current $I_a = \sqrt{3} * I_{ab}\angle -30^0\ A$

$I_a = \sqrt{3} * 24.5\angle -75^0\ A$

$I_a = 42.42\angle -75^0\ A$

$\theta$ = the angle between the phase current and voltage=45 $^0$

(i) Total 3-phase power, using equation 40:

$$P_T = \sqrt{3} * V_l * I_l * \cos(\theta) = \sqrt{3} * 173.2 * 42.42 * \cos(45) = 9000W$$

(ii) Total Reactive power using equation 41:

$$Q_T = \sqrt{3} * V_l * I_l * \sin(\theta) = \sqrt{3} * 173.2 * 42.42 * \sin(45) = 9000 Vars$$

(iii) Total Apparent power using equation 42:

$$S_T = \sqrt{3} * V_l * I_l = \sqrt{3} * 173.2 * 42.42 = 12{,}726\ VA$$

Compare the results of examples 2& 3.
The power consumption of the $\Delta$-connected load is 3 times the power requirements of the Y-connected load.

## 2.3 Summary

- The expression for power in n balanced "Y" circuits is give by

  (i) Total average power
  $P_T = \sqrt{3} * V_l * I_l * \cos(\theta)$ **(44)**

  (ii) Total Reactive power
  $Q_T = \sqrt{3} * V_l * I_l * \sin(\theta)$ (45)

  (iii) Total Apparent power

  $S_T = \sqrt{3} * V_l * I_l$ (46)

- The expression for power in n balanced " $\Delta$ " circuits is give by

  (i) Total average power
  $P_T = \sqrt{3} * V_l * I_l * \cos(\theta)$ **(47)**

  (ii) Total Reactive power
  $Q_T = \sqrt{3} * V_l * I_l * \sin(\theta)$ (48)

  (iii) Total Apparent power

  $S_T = \sqrt{3} * V_l * I_l$ (49)

The equations for the power expressions in Y-connected load and the $\Delta$-connected load are identical.
The equations for the power expressions in Y-connected load and the $\Delta$-connected load are identical.

But power values are not equal for the same load when connected in Y or $\Delta$ connection.
But power values are not equal for the same load when connected in Y or $\Delta$ connection.

## 3. Power Measurement in 3-phase circuits

Power is measured by an instrument called wattmeter. It has a voltage coil and a current coil.
Power is measured by an instrument called wattmeter. It has a voltage coil and a current coil.

The wattmeter deflection is a function of the voltage (V) and the current (I) and the angle $\theta$ between the voltage and the current.
The wattmeter deflection is a function of the voltage (V) and the current (I) and the angle $\theta$ between the voltage and the current.

The voltage V is the voltage across the voltage coil.

The voltage V is the voltage across the voltage coil

The current I is the current through the current coil.

The current I is the current through the current coil.

The current coil is in series with the load while the voltage coil is connected across the load.

The current coil is in series with the load while the voltage coil is connected across the load.

The voltage coil is connected by the phase and the neutral wire.

The voltage coil is connected by the phase and the neutral wire.

The power in a 3 phase circuit can be measured by inserting a wattmeter in each phase.

The power in a 3 phase circuit can be measured by inserting a wattmeter in each phase

The voltage coil is connected by the phase and the neutral wire.

The voltage coil is connected by the phase and the neutral wire.

The total power is found by computing the sum of the three meters.

The total power is found by computing the sum of the three meters.

This arrangement is quite suitable for Y-connected 4-wire systems

This arrangement is quite suitable for Y-connected 4-wire systems

It does not work for 3-wire Y-connected or Delta connected systems.

It does not work for 3-wire Y-connected or Delta connected systems.

The absence of the neutral wire of the Y-connection and the inaccessibility of the $\Delta$ phase are reasons to look for another arrangement.

The absence of the neutral wire of the Y-connection and the inaccessibility of the $\Delta$ phase are reasons to look for another arrangement.

This arrangement is called the two wattmeter method and is shown in figure 3.

This arrangement is called the two wattmeter method and is shown in figure 3.

Only two wattmeters are used.

Only two wattmeters are used.

The current coils of each of the wattmters are connected in series with one of the phase.

The current coils of each of the wattmters are connected in series with one of the phase.

Each of the voltage coils is connected across two phases with a common at the central phase.

Each of the voltage coils is connected across two phases with a common at the central phase.

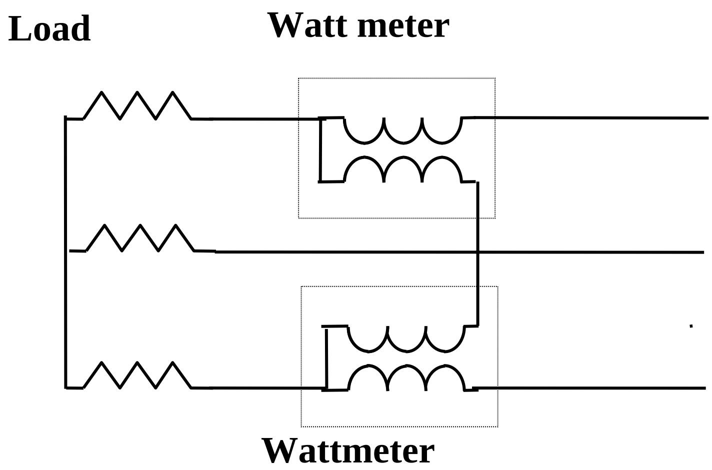


Figure 3 Two-Wattmeter Mthod for Measuring Three Phase Power

We will now explain how the algebraic sum of the wattmeter readings adds to the total load power

We will now explain how the algebraic sum of the wattmeter readings adds to the total load power
Let us make the following definitions:
Let us make the following definitions:

The upper wattmeter is called meter1 and its power is referred to as $W_1$.
The upper wattmeter is called meter1 and its power is referred to as $W_1$.
The lower wattmeter is called meter2 and its power is referred to as $W_2$.
The lower wattmeter is called meter2 and its power is referred to as $W_2$.

Current coil of wattmeter1 is connected through line a and it records the line current Ia.
Current coil of wattmeter1 is connected through line a and it records the line current Ia.

Current coil of wattmeter2 is connected through line c and it records line current Ic.

Current coil of wattmeter2 is connected through line c and it records line current Ic.

Voltage coil of wattmeter 1 is connected across lines a and b. It records Vab.
Voltage coil of wattmeter 1 is connected across lines a and b. It records Vab.

Voltage coil of wattmeter 2 is connected across lines c and b. It records Vcb.
Voltage coil of wattmeter 2 is connected across lines c and b. It records Vcb.

The power recorded by both wattmeters is given by
The power recorded by both wattmeters is given by

$$W_1 = V_{ab} * I_a \cos(\phi_a) \qquad (50)$$

$$W_2 = V_{cb} * I_a \cos(\phi_c) \qquad (51)$$

Where
$\phi$ is the angle between the phasor of $V_{ab}$ and the current $I_a$.
$\phi$ is the angle between the phasor of $V_{cb}$ and the current $I_c$.

Consider either Y-connected or $\Delta$-connected load and refer to figures 4 and 5 respectively
Consider either Y-connected or $\Delta$-connected load and refer to figures 4 and 5 respectively

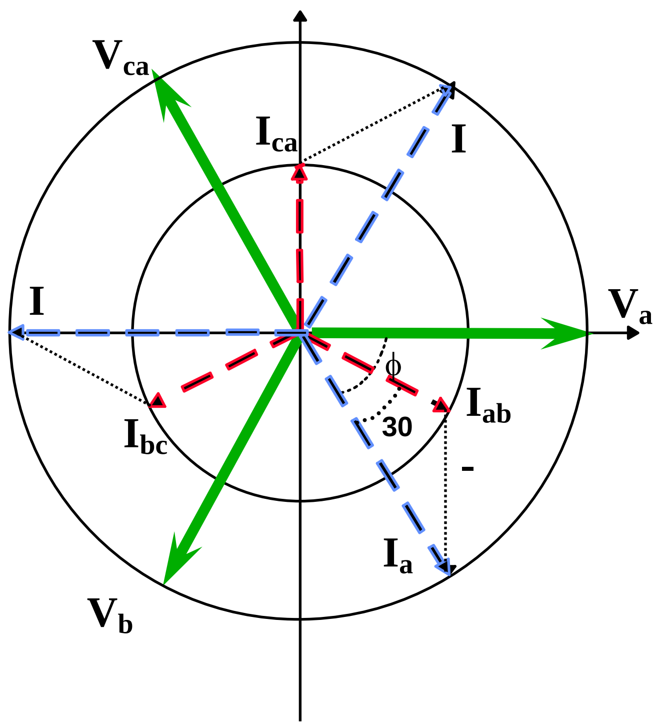


Figure 5 Phasor Diagram for Delta connected load.

The angle between the line voltage Vab and and $I_a$ is $(30+\theta)$.
The angle between the line voltage Vab and and $I_a$ is $(30+\theta)$.

The angle between the line voltage Vcb and and $I_c$ is $(30-\theta)$.
The angle between the line voltage Vcb and and $I_c$ is $(30-\theta)$.

For a balanced system
For a balanced system

$V_{ab}=V_{cb}=V_L$
$I_a = I_c = I_L$

Equations 64 and 65 can be written as follows:
Equations 64 and 65 can be written as follows:

$$W_1 = V_L * I_L \cos(30+\theta) \quad (52)$$

$$W_2 = V_L * I_L \cos(30-\theta) \quad (53)$$

The following points are observed.

(i) If the load is resistive $(\theta = 0)$, $W_2=W_1$
If the load is resistive $(\theta = 0)$, $W_2=W_1$

(ii) If the load is inductive $(\theta > 0)$ ; $W_2>W_1$
If the load is inductive $(\theta > 0)$ ; $W_2>W_1$

(iii) If the load is capacitive$(\theta < 0)$ ; $W_2<W_1$

If the load is capacitive$(\theta < 0)$ ; $W_2<W_1$

The total recorded power is
The total recorded power is

$P_T= W_1+W_2$
$$W_1+W_2 = V_L * I_L(\cos(30+\theta)+\cos(30-\theta)) \quad (54)$$

Expand equation 54 and substitute for the substituting the values of sine and cosine the angle 30 will yield the following relationship.

Expand equation 62 and substitute for the substituting the values of sine and cosine the angle 30 will yield the following relationship.

$$W_1+W_2 = \sqrt{3} * V_L * I_L * \cos(\theta) \quad (55)$$

The sum of the two wattmeters gives the total 3 phase power as expressed in 47
The sum of the two wattmeters gives the total 3 phase power as expressed in 47

$$P_T=W_1+W_2 \quad (56)$$

Moreover and using the same procedure, one can write the following equation:
Moreover and using the same procedure, one can write the following equation:

$$W_2-W_1 = * V_L * I_L * \sin(\theta) \quad (57)$$

Use the definition of the total reactive power (48) and rearrange equation 71.
Use the definition of the total reactive power (48) and rearrange equation 71.

$$Q_T = \sqrt{3} * (W_2 - W_1) \quad (58)$$

Dividing equation 72 by 70 will give the tangent of the power factor angle
Dividing equation 72 by 70 will give the tangent of the power factor angle

$$\tan(\theta) = \frac{Q_T}{P_T} = \sqrt{3} * (\frac{W_2 - W}{W_2 + W_1}) \qquad (59)$$

**Example 4**

Two wattmeters were used to measure power absorbed by Y-connected load with a line voltage of 208-V. The recording of the two meters are $W_1$=-280 W and $W_2$=400 W. Determine

(i) total average power
(ii) total reactive power
(iii) The power factor
(iv) The line current
(v) The load impedance per phase

Solution

(i) The total average power is

$P_T = W_1 + W_2$ =-280+400=120 W

(ii) The reactive power is

$Q_T = \sqrt{3} * (W_2 - W_1) = \sqrt{3} * (400 - (-280)) = 1177.76\ Vars$

(iii) The power angle is

$$\theta = \tan^{-1}(\frac{Q_T}{P_T}) = \tan^{-1}(\frac{1177.76}{120}) = 84.18^0$$

Power factor = $\cos(\theta) = \cos(84.18) = 0.1014$

(iv) the line current

$$I_L = \frac{P_T}{(\sqrt{3} * V_L * (\cos(\theta))} = \frac{120}{\sqrt{3} * 208 * 0.1014} = 3.28\,A$$

(v) The load impedance

$$Z_P = \frac{V_p}{I_p}$$

For Y-connected

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120V$$

$$I_p = I_L = 3.28\,A$$

$$Z_P = \frac{V_p}{I_p} = \frac{120}{3.28} = 36.61\ \Omega$$

The angle of the load impedance is the same as the power factor angle
The angle of the load impedance is the same as the power factor angle

Therefore the load impedance is

$$Z_P = 36.61\angle 84.18\ \Omega$$