Lecture Notes
EEE 360

Transformer (Part 1: Single Phase)

Read Chapter : **4.1-4.6**

Transformer

Magnetic circuit analysis

- **The magnetic circuit consists of a laminated iron core and a winding.**
- **The AC current in the winding generates an AC magnetic flux in the core.**
- **The magnetic field is calculated by Ampere's law:**

$$\mathcal{F} = I\,N = H\,L_c$$

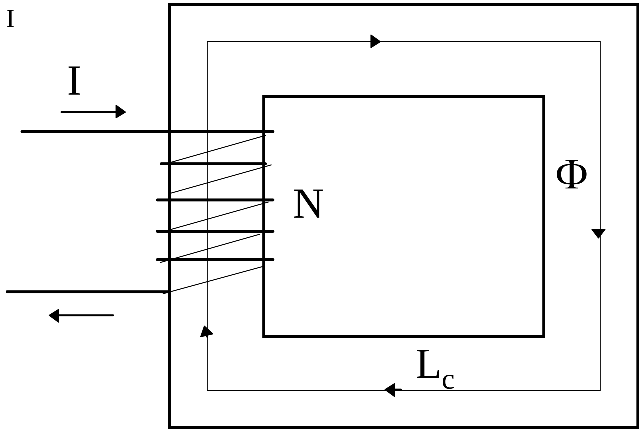
where: $\mathcal{F}$ is the magnetomotive force

N is the number of turns

I is the current

H is the magnetic field

L_c is he magnetic path length



Transformer

Magnetic circuit analysis

- Magnetic flux density (Weber/m^2 or Teslas)

  $B = \mu H = \mu I N / L_c$

where:

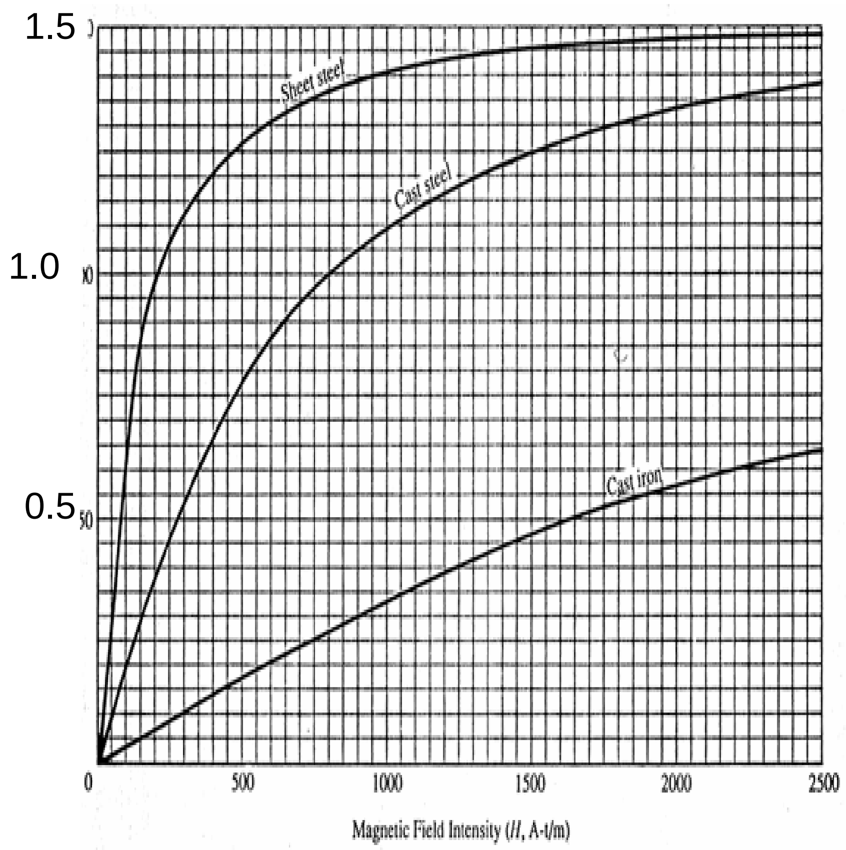
  - μ is the permeability in H/m. $\mu = \mu_o \mu_r$
  - $\mu_o = 4 \pi 10^{-7}$ H/m free space permeability (air)
  - μ_r relative permeability (air $\mu_r = 1$, and iron $\mu_r = 5000\text{-}8000$)
  - The actual value of μ_r is determined from the B - H magnetization curve of the magnetic material.

Transformer

Magnetic circuit analysis

- **The magnetic circuits are built with laminated core.**
- **The core is made out of silicon iron sheets.**
- **B - H curves of three magnetic materials are shown in the figure.**
- **The permeability of the core is the slope of the B - H curve.**
- **The operating range is below the saturation (knee) of the curve. This region is more or less linear.**

B - H Curves. (B in teslas, H in amp/m)



Transformer

Magnetic circuit analysis

- The next step of the analysis is the calculation of the magnetic flux.
- The flux is measured by Weber
- The majority of the flux is in the iron core. Small leakage flux may appears in the air, which is neglected here.
- The flux is:

$$\phi_m = B\ A = \mu\ H\ A = I\ \mu\ N A / L_c$$

Where: B is the flux density in Tesla

A is the cross section of the iron core

Transformer

Magnetic circuit analysis

- **The sinusoidal AC flux induces voltage in the coil (Faraday's Law)**
- **This voltage is equal to the supply voltage if the ohmic voltage drop is neglected.**
- **The induced voltage is:** $e(t) = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$
- **The equation for the sinusoidal flux is:** $\phi(t) = \phi_m \sin(\omega t)$
- **Therefore, the expression for the induced voltage becomes:**

$$e(t) = N\frac{d\phi}{dt} = N\,\phi_m\,\omega\cos(\omega t)$$

- **The rms value of the induced voltage is:**

$$V = \frac{N \cdot \phi_m \cdot \omega}{\sqrt{2}} = 4.44 \cdot N \cdot f \cdot \phi_m$$

where: V is in volts, f = 60 Hz, and ϕ_m in Weber

Transformer

Magnetic circuit analysis

- **The substitution of the $\phi = \mu \; I (N A / L_c)$ in the induced voltage equation yields:**

$$e = N\frac{d\phi}{dt} = N^2\frac{\mu \cdot A}{L_c}\frac{di}{dt} = L\frac{di}{dt}$$

where $L = \mu \cdot \frac{N^2 \cdot A}{L_c}$

is the inductance

- **The energy in the magnetic field is integral of the incoming electric power:**

$$W = \int i \cdot e \cdot dt = \int i \cdot L \cdot \frac{di}{dt} \cdot dt = L\int_0^I i \cdot di = \frac{1}{2} \cdot L \cdot I^2$$

- **The voltage induced in a second coil placed on the core is:**

$$e_2 = N_2 \cdot \frac{d\phi}{dt} = N_2 \cdot N_1 \cdot \frac{\mu \cdot A}{L_c} \cdot \frac{di}{dt} = L_{1,2} \cdot \frac{di}{dt} \qquad L_{1,2} = \mu_0 \frac{N_1 N_2}{L_c} A$$

Test 1

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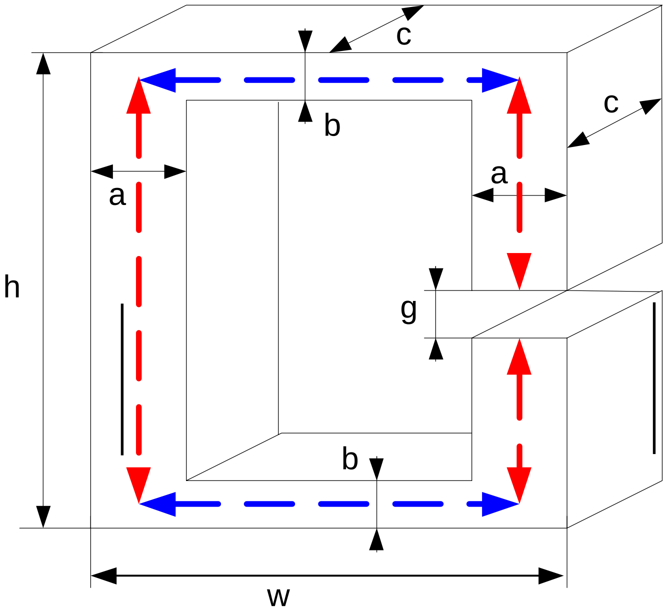
Transformer

Magnetic Circuit Analyses

- The arrangement of an inductor with air gap is shown on the Figure
- A coil is placed on the iron-core.
- The magnetic flux density in the air gap is B_{gap}. when the current is I_m.



Transformer

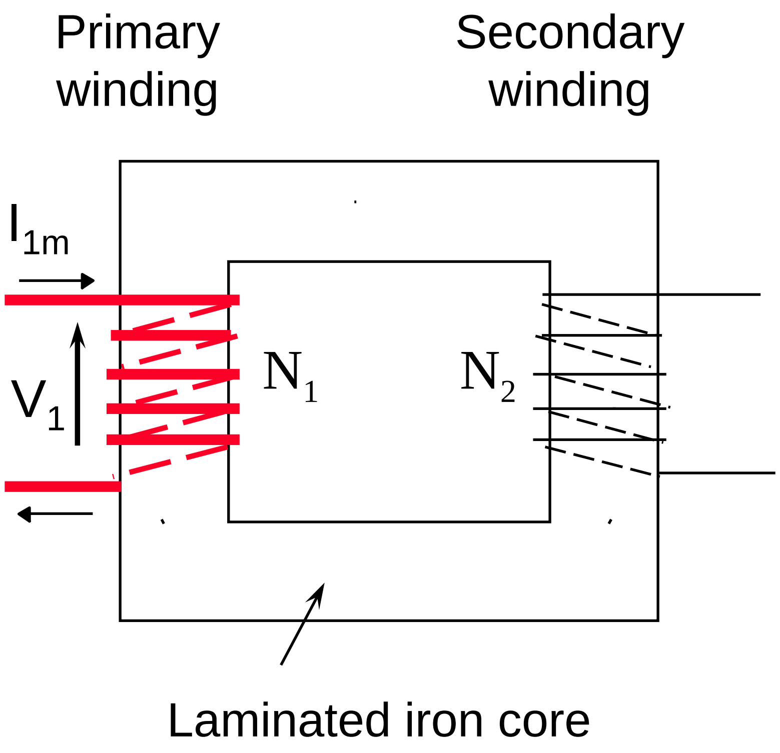
Calculate

- **1) Magnetic flux and the flux density in the different section of the iron core**
- **2) Determine the magnetic field intensity (H) in the different section of the core**
- **3) Calculate the number of turns needed to maintain the flux if the current is Im**
- **4) Calculate the inductance when the iron-core is neglected or considered.**
- **5) Calculate the induced voltage in a coil, with 150 turns, if it is placed on the iron-core Assume that the frequency is 60 Hz**
- **6). Determine the required supply voltage if the resistance is neglected**

Transformer

Ideal Transformer

- **The transformer has laminated iron-core and a primary and secondary winding**
- **The windings resistance and leakage flux are zero.**
- **The primary winding is supplied by a sinusoidal voltage V_1.**
- **The V_1 voltage drives a magnetizing current through the winding. I_{1m}**



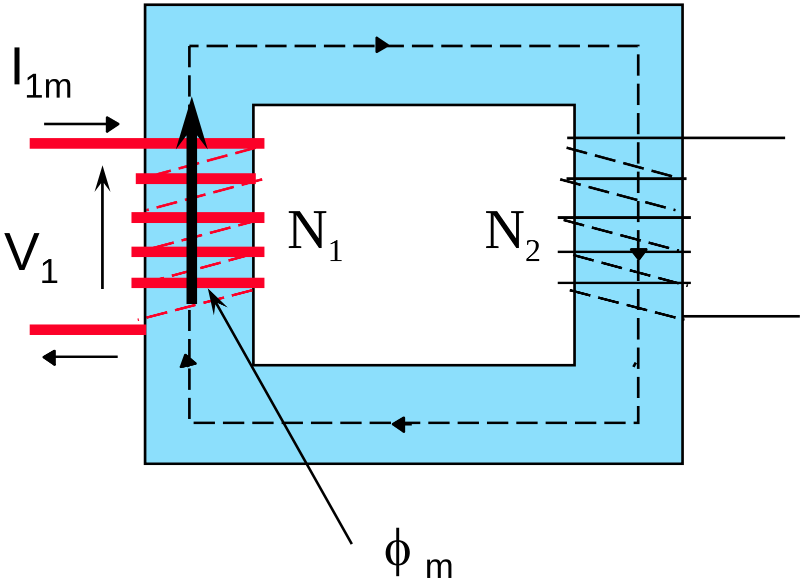
Transformer

Ideal Transformer

- Magnetizing current I_{1m} generates a magnetization flux ϕ_m in the iron core.
- The flux changes more or less in sinusoidal form.
- The relation between the flux and voltage is :

$$V_1 = N_1 \cdot \frac{d\phi_m}{dt} = 4.44 \cdot f \cdot N_1 \cdot \phi_{max}$$

Flux generation



Transformer

Ideal Transformer

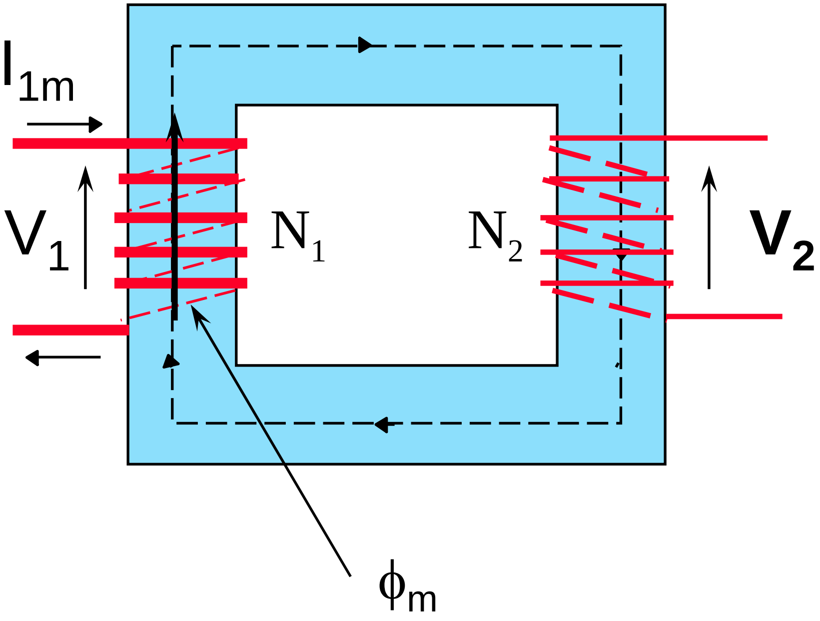
- **The ac flux links to the secondary winding.**
- **The flux change ($d\phi_m$ /dt) induces a sinusoidal voltage V_2 in the secondary winding.**
- **The induced voltage is :**

$$V_2 = N_2 \cdot \frac{d\phi_m}{dt} = 4.44 \cdot f \cdot N_2 \cdot \phi_{max}$$

- **The ratio of the primary and secondary voltages is called the turn ratio:**

$a = V_1/V_2 = N_1/N_2$

Voltage generation



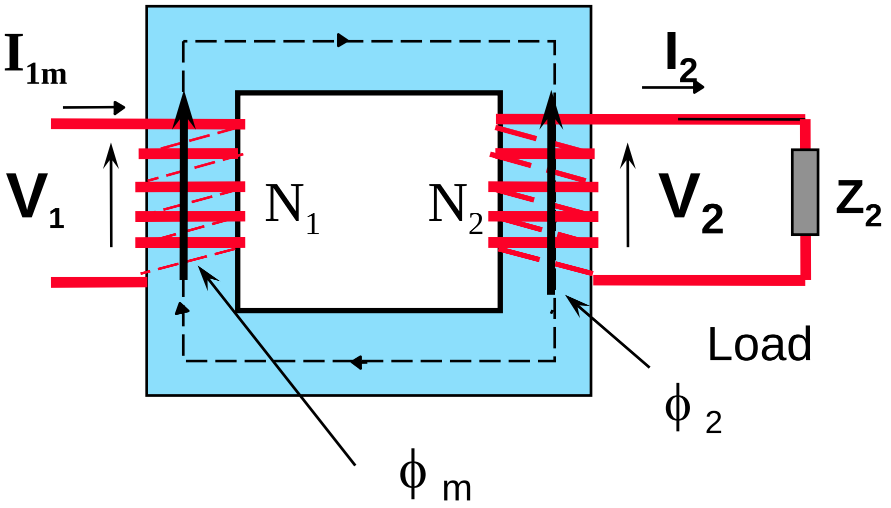
Transformer

Ideal Transformer

- **A load impedance Z_2 is connected to the secondary.**
- **The secondary voltage V_2 drives a load current through Z_2 The current is** :

  $I_2 = V_2 / Z_2$
- **The load current generates a flux ϕ_2 that opposes the magnetization flux ϕ_m.**

Transformer loaded

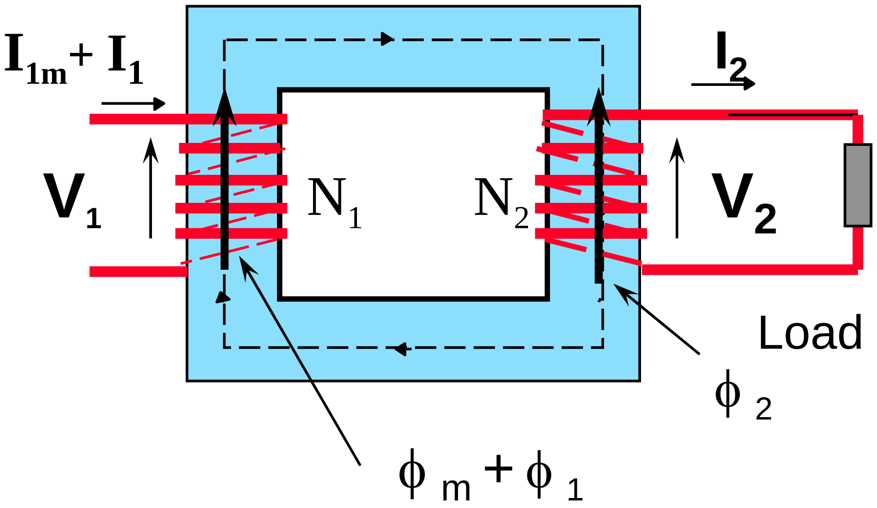


Transformer

Ideal Transformer

- **The load flux ϕ_2 induces a voltage in the primary winding that opposes the supply voltage.**
- **The supply voltage is constant. Therefore the reduction of the induced voltage increases the primary current. ($I_{1m}+ I_1$).**
- **The I_1 current generates a flux ϕ_1 that balances and equalizes the flux ϕ_2 generated by the secondary current I_2.**

Transformer loaded



Transformer

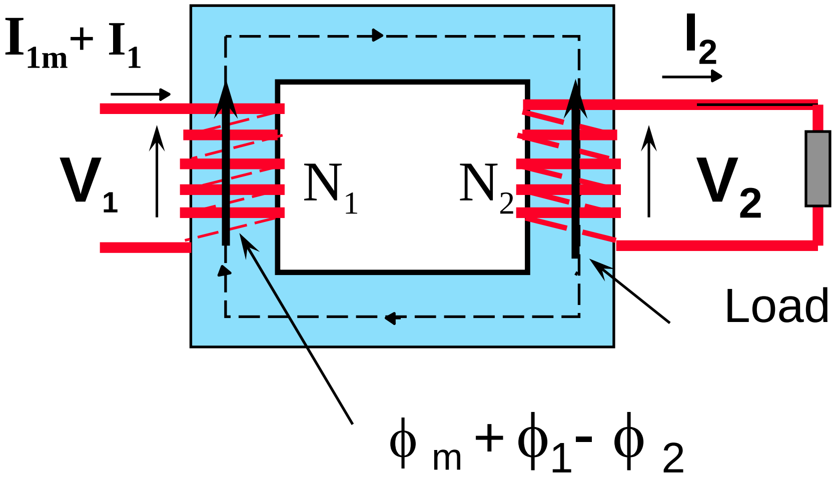
Ideal Transformer

- **The flux equalization produces the following effects:**
  - **The core flux ϕ_m remains constant and independent from the load.**
  - **The primary magneto-motive force $\mathcal{F}_1$ is equal to the secondary magnetomotive force $\mathcal{F}_2$. Therefore:**

    $I_1 N_1 = I_2 N_2$

    This equation assumes that the magnetizing current I_m is negligible small

Transformer loaded



Transformer

Ideal Transformer

- **The losses are zero in an ideal transformer. Therefore the input power (VA) is equal to the output power (VA).**

  $I_1 V_1 = I_2 V_2$

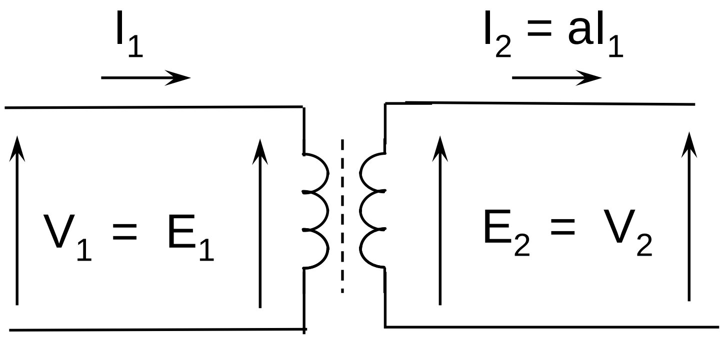
- **The voltage and current relations are:**

  $a = V_1 / V_2 = I_2 / I_1$ **or**

  $V_2 = V_1 / a$ **and** $I_2 = I_1 a$

- **If a transformer increases the voltage, the current decreases and viceversa.**

Equivalent Circuit



$E_2 = E_1 / a$

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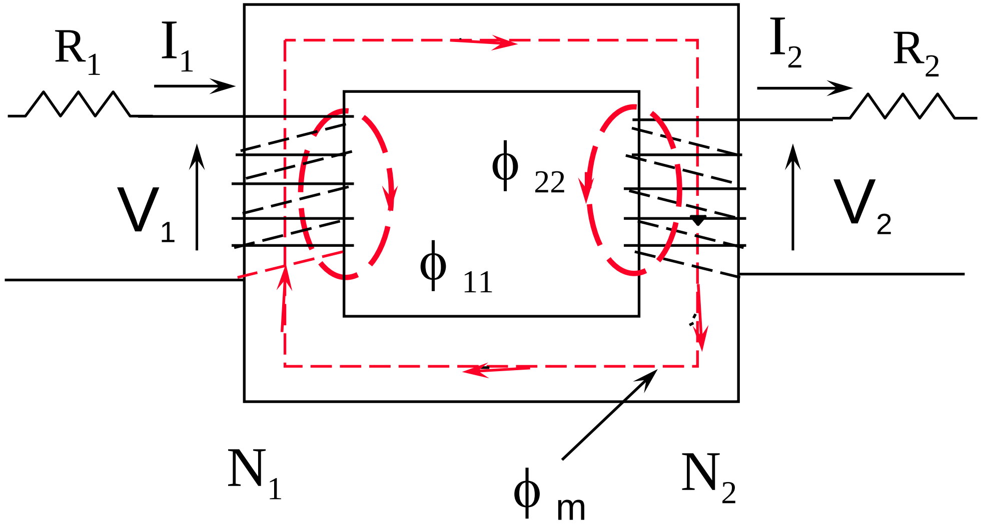
Transformer

Actual Transformer

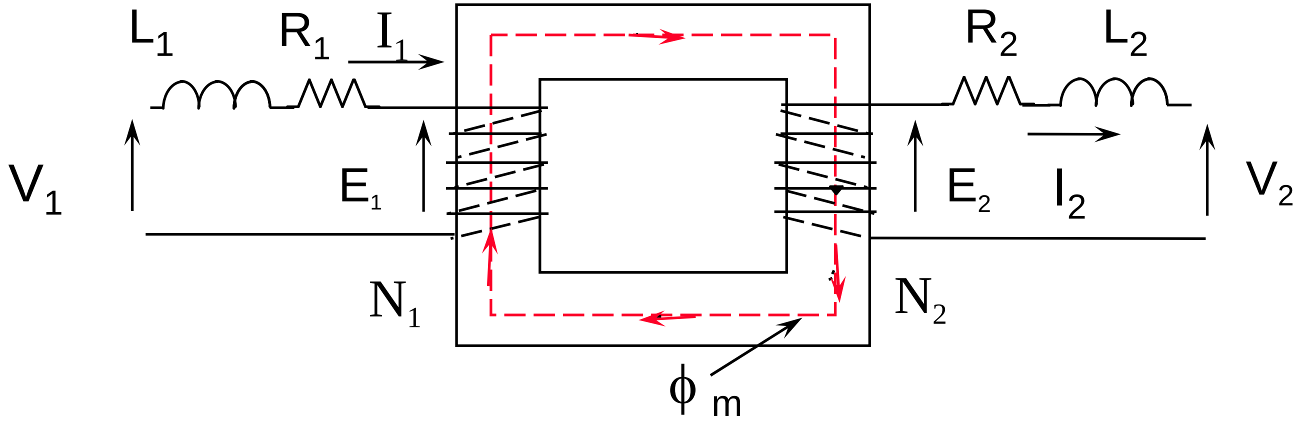
- **The actual transformer windings have resistances R_1 and R_2, which are removed from the windings and placed in series with them.**
- **Part of the primary current generated flux will not link the secondary winding. This flux is the primary leakage flux ϕ_{11}.**
- **Part of the secondary current generated flux will not link the primary winding. This flux is the secondary leakage flux ϕ_{22}.**



Transformer

Actual Transformer

- **The flux linking the primary winding is:** $\phi_1 = \phi_m + \phi_{11} = \phi_m + \frac{I_1 L_1}{N_1}$
- **The flux linking the secondary winding is:** $\phi_2 = \phi_m + \phi_{22} = \phi_m + \frac{I_2 L_2}{N_1}$
- Φ_{11} **and** Φ_{22} **can be replaced by equivalent inductance** L_1 **and** L_2 **respectively**

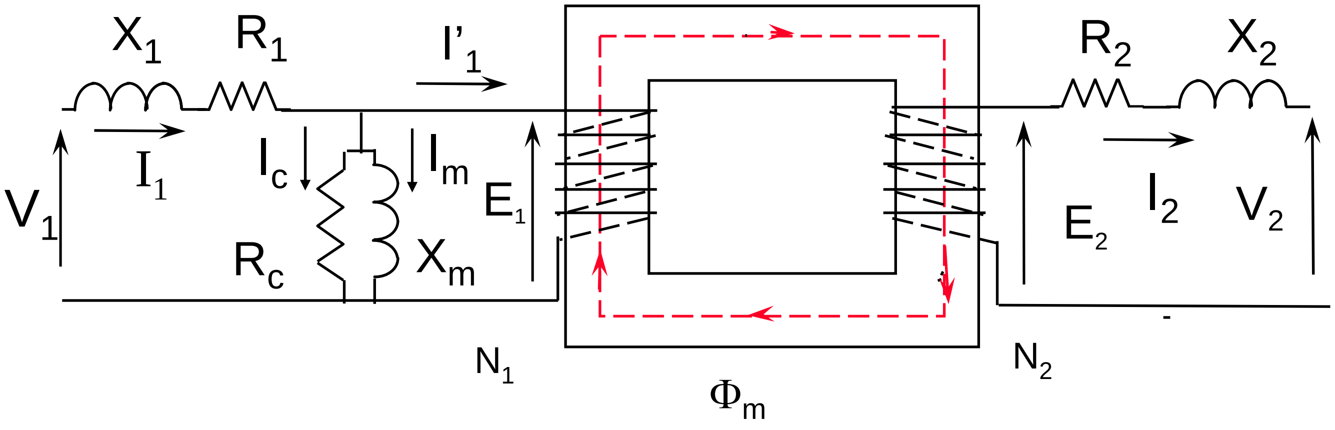


Transformer

Actual Transformer

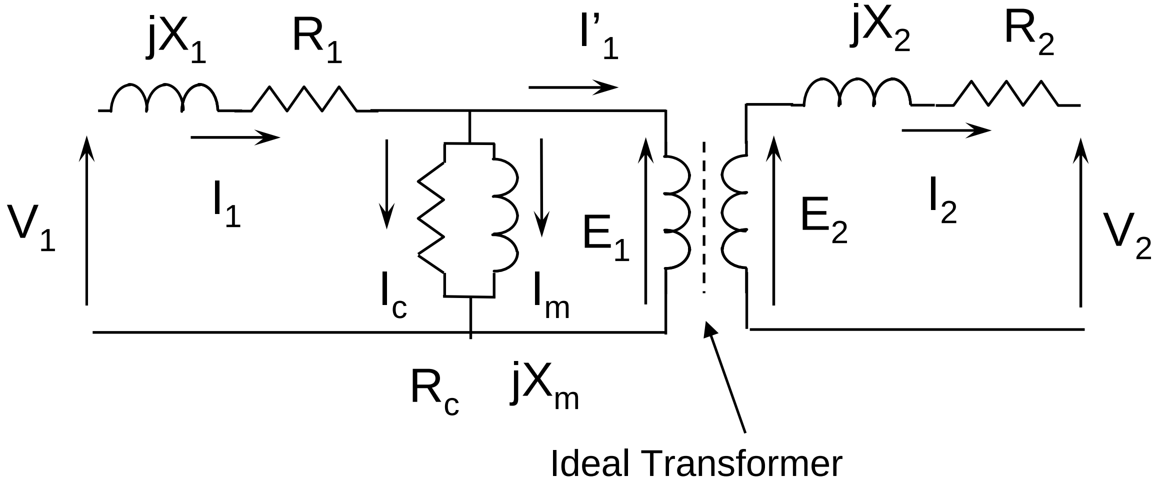
- In a real transformer the iron core permeability is not infinite and the magnetizing current is not negligible. The iron core is represented by a magnetizing reactance X_m

- Hysteresis and eddy currents cause iron losses. These losses are represented by a resistance R_c which is connected in parallel with X_m



Transformer

Actual Transformer. Equivalent circuit



Transformer

Simplification of equivalent circuit

- **The equations for an ideal transformer**

$E_1 = a E_2$ $\qquad$ $I_1 = I_2 / a$

The division of the two equation result in

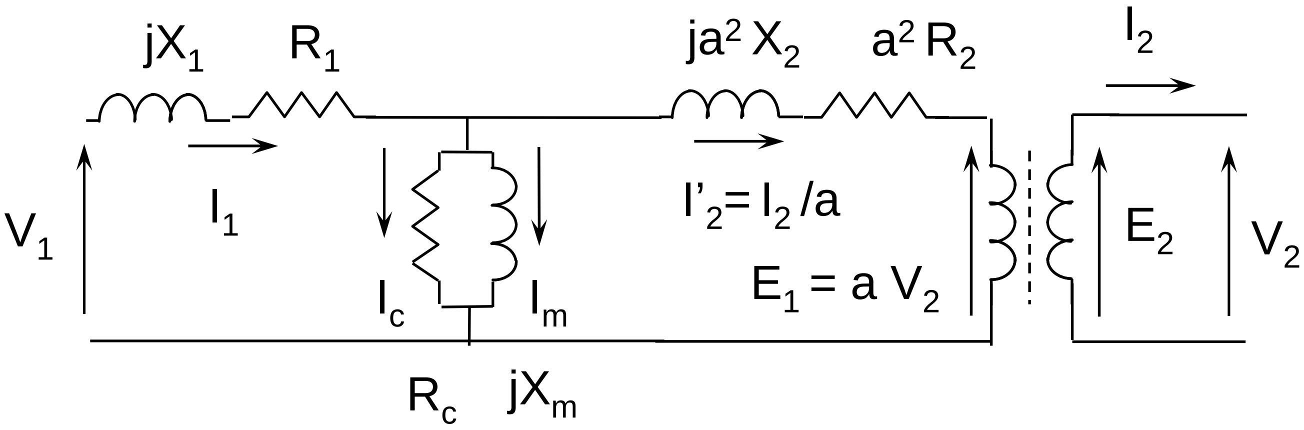
$$\frac{E_1}{I_1} = a^2 \frac{E_2}{I_2} \qquad Z_1 = a^2 Z_2$$

- **An impedance can be transferred from one side to the other by multiplying by the square of the turns ratio.**

Transformer

Simplification of equivalent circuit

The transfer of the impedances from the secondary to the primary results in :

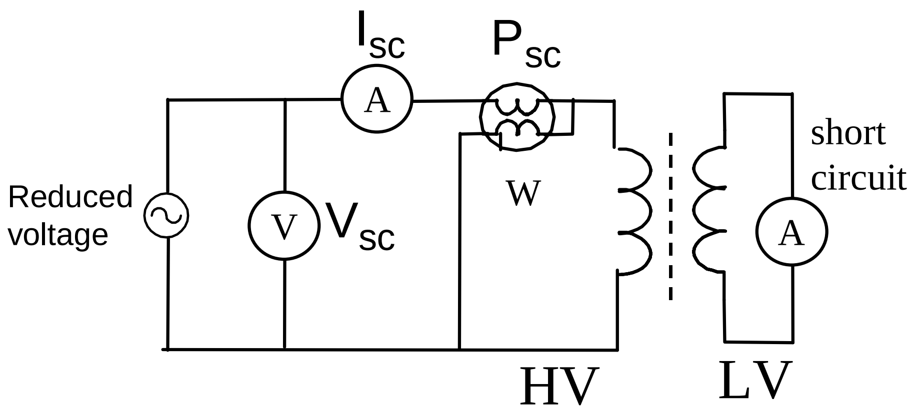


Transformer

Equivalent circuit parameters

- **The series impedance of a transformer is calculated from a short circuit test.**
- **The low voltage side (LV) is short-circuited and the high voltage (HV) is supplied by a reduced voltage which drives rated current through the transformer.**
- **The voltage, current, and input power are measured.**

Short- circuit test



Transformer

Equivalent circuit parameters

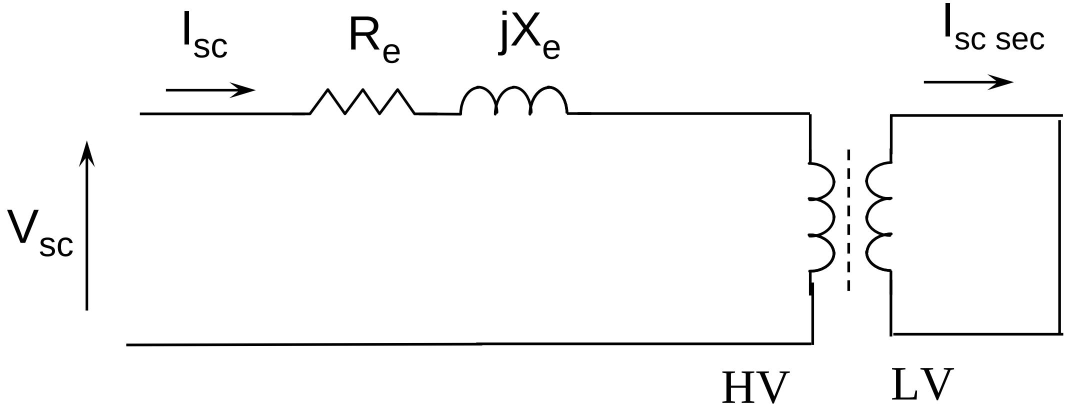
- **The short circuit test gives the current I_{sc}, the supply voltage V_{sc} and the power loss P_{sc}.**
- **The equivalent circuit shows that the series impedance can be calculated from this data.**

- **The series impedance calculation is:**

$Z_e = V_{sc} / I_{sc}$ and $R_e = P_{sc} / I_{sc}^2$

Therefore, the equivalent reactance is:

$$X_e = \sqrt{Z_e^2 - R_e^2}$$

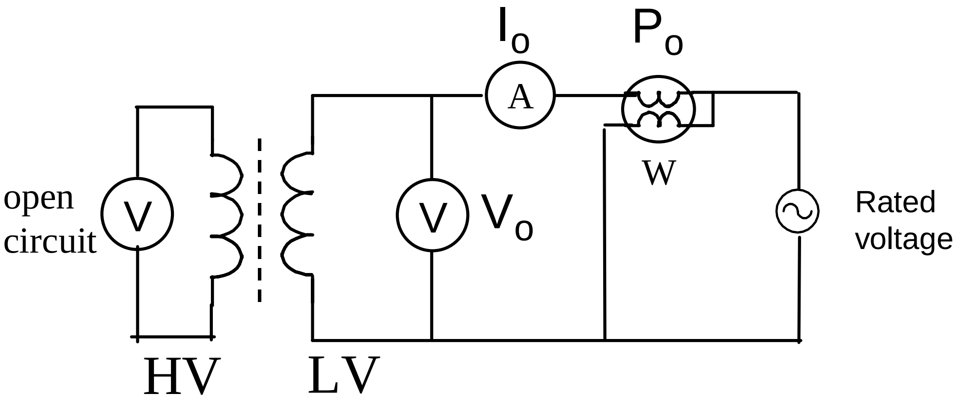


Transformer

Equivalent circuit parameters

- **The magnetizing impedance of a transformer is calculated from the open-circuit test.**
- **The high voltage side (HV) is open and the low voltage side (LV) is supplied by the rated voltage, which drives magnetizing current through the transformer.**
- **The voltages, current and impute power are measured.**

Open - circuit test



Transformer

Equivalent circuit parameters

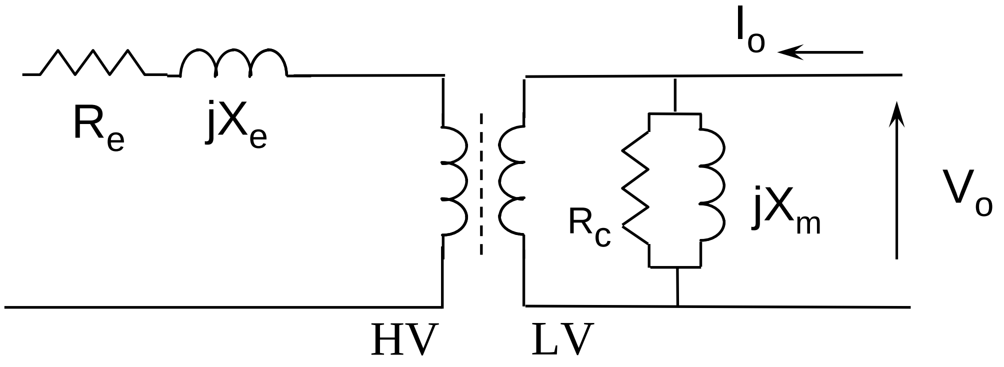
- **The open-circuit test gives the magnetizing branch current I_o, the supply voltage V_0 and the iron loss P_c.**
- **The equivalent circuit shows that the magnetizing impedance can be calculated from this data.**

- **If the series impedance is negligible, the magnetizing impedance is:**

$$R_c = V_0^2 / P_0$$

as $S_o = V_0 I_0$ and

then $Q_m = \sqrt{S_0^2 - P_0^2}$



$$X_m = \frac{V_0^2}{Q_0}$$

Transformer

Questions to ponder.

- **Why the discovery of the transformer accelerated the development of the use of electricity ?**
- **Why the iron or magnetization losses are more important than the losses caused by the winding resistance ?**
- **Why the transformer has to be cooled ? How it is done ?**
- **What is the connection of the transformer that supplies your house and where is it ?**

Transformer

A 25 kVA, 2400 V / 240 V single phase transformer was tested.

In the short-circuit tests, the HV side was shorted and the parameters were measured at the LV side.

In the open-circuit test, the LV side was opened and the parameters were measured at the HV side.

The results of the tests are :

- **Short-circuit test (HV shorted) Vs = 80 volt, Is = 400 amp, PS = 800 watt**
- **Open-circuit test (LV open) Vo = 2400 volt, Io = 2 amp, Po = 300 watt**

Transformer

a) Calculate the transformer parameters.

b) Draw the simplified equivalent circuit

c) Calculate and plot the transformer voltage regulation vs. load and determine the load when the regulation is 5%

d) Calculate and plot the efficiency vs. load. Determine the load when the efficiency is maximum and the maximum efficiency

Assume: power factor 0.8 lagging and load voltage = rated voltage