

EE 205 Dr. A. Zidouri

Electric Circuits II

Two-Port Circuits
Two-Port Parameters

Lecture #42

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The material to be covered in this lecture is as follows:

- Introduction to two-port circuits
- The Terminal Equations
- The Two-Port z-parameters
- The Two-Port y-parameters

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After finishing this lecture you should be able to:

- Understand the Importance of Two-Port Circuits
- Relate the Current and Voltage at One Port to the Current and Voltage at the Other Port.
- Determine the Two-Port z-parameters
- Determine the Two-Port y-parameters

Introduction to Two-Port Circuits

- ✚ In analyzing some electrical systems, focusing on two pairs of terminal is convenient.
- ✚ Often, a signal is fed into one pair of terminals and then after being processed, is extracted at a second pair of terminals.
- ✚ The terminal pairs represent the points where signals are either fed in or extracted. They are referred to as **ports** of the system.
- ✚ Fig. 42-1 illustrates the basic two-port building block.
- ✚ Use of this building block is subject to several restrictions:
 - There can be no energy stored within the circuit
 - There can be no independent sources within the circuit
 - The current into the port must equal the current out of the port
 - All external connections must be made to either the input port or output port, no connections are allowed between the ports.

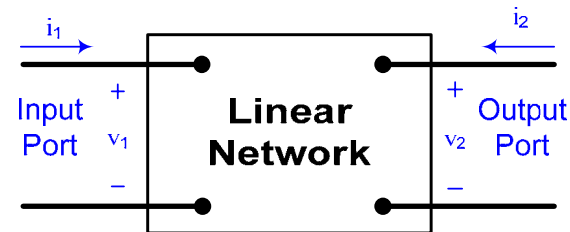


Fig. 42-1 The Two-Port Building Block

- ✚ The fundamental principle underlying two-port modeling of a system is that only the terminal variables (i_1 , v_1 , i_2 , and v_2) are of interest.

The Terminal Equations

- In two-port network we are interested in relating the current and voltage at one port to the current and voltage at the other port. Fig. 42-1 shows the reference polarities of the terminal voltages and the reference directions of the terminal currents.
- Most general description is carried out in the s domain.
- We write all equations in the s domain, resistive networks and sinusoidal steady state solutions become special cases.
- Fig. 42-2 shows the basic building block in terms of the s-domain variables I_1 , V_1 , I_2 , and V_2 .

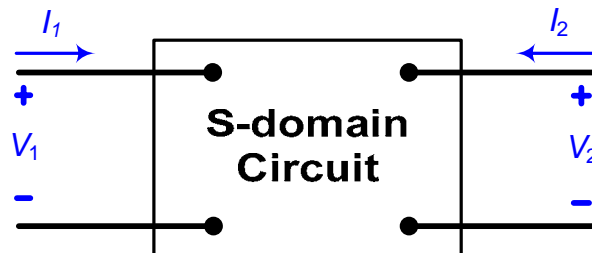


Fig. 42-2 The s-domain Two-Port Basic Building Block

- Out these four terminal variables, only two are independent. Thus we can describe a two-port network with just two simultaneous equations. However there are six ways in which to combine the four variables:

Impedance Parameters (**z-parameters**):

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned} \quad (42-1)$$

Admittance Parameters (**y-parameters**):

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned} \quad (42-2)$$

Hybrid Parameters (**h-parameters**):

$$\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned} \quad (42-3)$$

Inverse Hybrid Parameters (**g-parameters**):

$$\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \\ V_2 &= g_{21}V_1 + g_{22}I_2 \end{aligned} \quad (42-4)$$

Transmission Parameters (**a-parameters**):

$$\begin{aligned} V_1 &= a_{11}V_2 - a_{12}I_2 \\ I_1 &= a_{21}V_2 - a_{22}I_2 \end{aligned} \quad (42-5)$$

Inverse Transmission Parameters (**b-parameters**):

$$\begin{aligned} V_2 &= b_{11}V_1 - b_{12}I_1 \\ I_2 &= b_{21}V_1 - b_{22}I_1 \end{aligned} \quad (42-6)$$

- ✚ These six sets of equations may also be considered as three pairs of mutually inverse relations.
- ✚ The coefficients of the variables are called the **parameters** of the two-port circuit. We refer to the z-parameters, y-parameters, a-parameters, b-parameters, h-parameters and g-parameters of the network.

The Two-Port Parameters

z-parameters:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

or in matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (42-7)$$

The values of the parameters can be evaluated by setting $I_1=0$ (input port open-circuited) or $I_2=0$ (output port open-circuited). Thus,

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}, \quad z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad (42-8)$$

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}, \quad z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

The z-parameters are also called the open-circuit impedance parameters:

- z_{11} = Open-circuit input impedance
- z_{12} = Open-circuit transfer impedance from port 1 to port 2
- z_{21} = Open-circuit transfer impedance from port 2 to port 1
- z_{22} = Open-circuit output impedance

Example 42-1 illustrates the determination of the z-parameters for a resistive circuit.

Example 42-1

Find the z -parameters for a resistive circuit shown in Fig. 42-3

Solution:

To obtain z_{11} and z_{21} we connect a voltage V_1 (or a current source I_1) to port 1 with port 2 open circuited as in Fig. 42-4a.

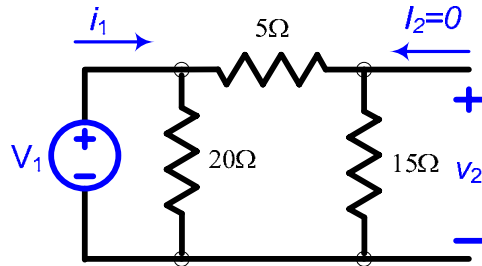


Fig. 42-4a Circuit for finding z_{11} and z_{21}

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{20 \times 20}{40} = 10\Omega,$$

When I_2 is zero, $V_2 = \frac{V_1}{15+5} \times 15 = 0.75V_1$ therefore $z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{0.75V_1 \times 20}{V_1/10} = 7.5\Omega$

To obtain z_{12} and z_{22} we connect a voltage V_2 (or a current source I_2) to port 2 with port 1 open circuited as in Fig. 42-4b.

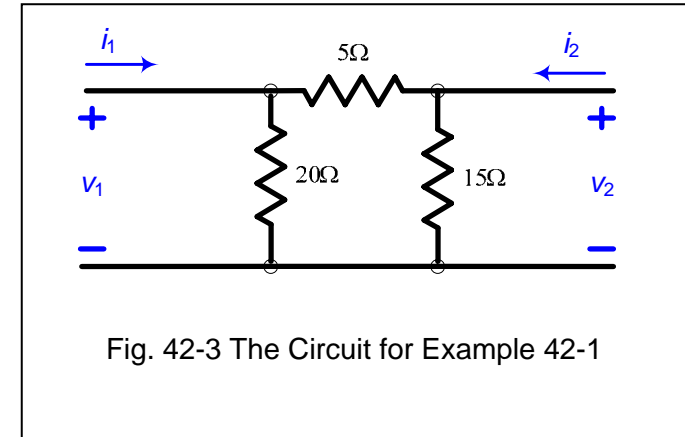


Fig. 42-3 The Circuit for Example 42-1

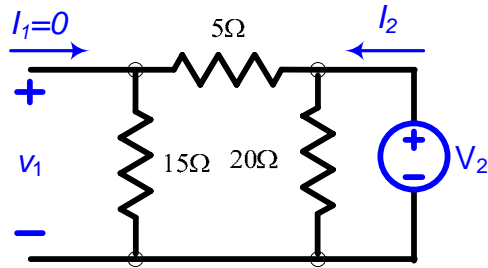


Fig. 42-4b Circuit for finding z_{12} and z_{22}

$$z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \frac{15 \times 20}{40} = 7.5 \Omega,$$

When I_1 is zero, $V_1 = \frac{V_2}{5+20}(20) = 0.8V_2$ and $I_2 = \frac{V_2}{9.375}$ hence $z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5 \Omega$

Note that each of these parameters is the ratio of a voltage to a current and therefore is an impedance with the dimension of ohms; this is why they are called **z**-parameters.

When $z_{11} = z_{22}$, the two-port network is said to be **symmetrical**.

When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($z_{12}=z_{21}$), and the two-port network is said to be **reciprocal**.

y-parameters:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

or in matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (42-9)$$

The values of the parameters can be evaluated by setting $V_1=0$ (input port short-circuited) or $V_2=0$ (output port short-circuited). Thus,

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}, y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} \quad (42-10)$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}, y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

The *y*-parameters are also called the short-circuit admittance parameters:

- y_{11} = Short-circuit input admittance
- y_{12} = Short -circuit transfer admittance from port 2 to port 1
- y_{21} = Short -circuit transfer admittance from port 1 to port 2
- y_{22} = Short -circuit output admittance

Example 42-2 illustrates the determination of the *y*-parameters for a resistive circuit.

Example 42-2

Obtain the y -parameters for the resistive circuit shown in Fig. 42-5

Solution:

To obtain y_{11} and y_{21} we connect a current I_1 (or a voltage source V_1) to input port 1 with output port 2 short circuited as in Fig. 42-6a.

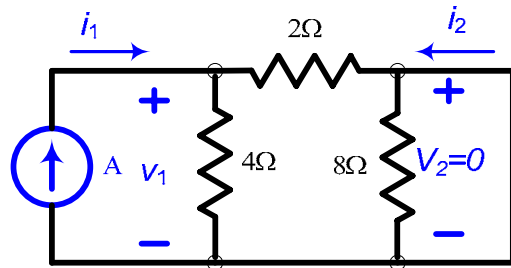


Fig. 42-6a Finding y_{11} and y_{21} for Example 42-2

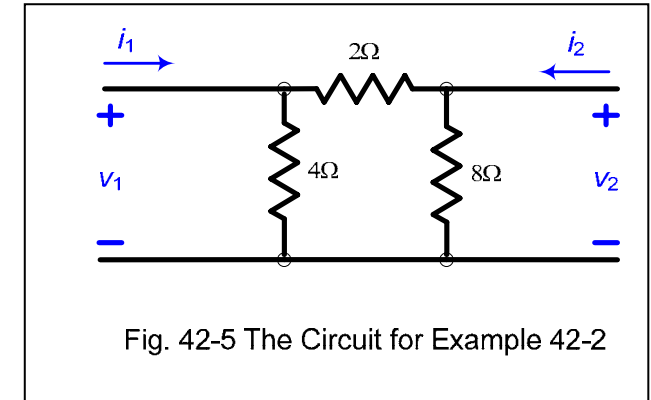


Fig. 42-5 The Circuit for Example 42-2

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \left. \frac{I_1}{I_1(4 \parallel 2)} \right|_{V_2=0} = \left. \frac{I_1}{\frac{4}{3}I_1} \right|_{V_2=0} = 0.75S,$$

$$\text{When } V_2 \text{ is zero, } -I_2 = \frac{4}{4+2}I_1 = \frac{2}{3}I_1 \text{ and } V_1 = \frac{4}{3}I_1 \text{ hence } y_{21} = \left. \frac{I_2}{V_1} \right|_{I_2=0} = \frac{-(\frac{2}{3})I_1}{(\frac{4}{3})I_1} \Big|_{I_2=0} = -0.5S$$

To obtain y_{12} and y_{22} we connect a current source I_2 (or a voltage source V_2) to port 2 with port 1 short circuited as in Fig. 42-6b.

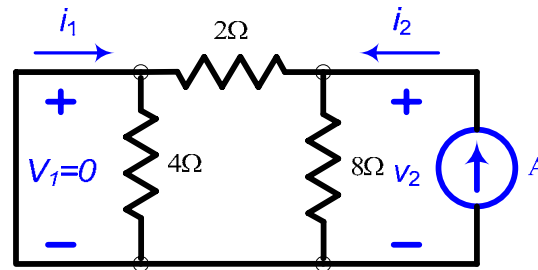


Fig. 42-6b Finding y_{12} and y_{22} for Example 42-2

$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{I_2}{I_2(8 \parallel 2)} \Big|_{V_2=0} = \frac{I_2}{\frac{8}{5}I_2} \Big|_{V_1=0} = 0.625S$$

When V_1 is zero, $-I_1 = \frac{I_2}{8+2}(8) = 0.8I_2$ and $V_2 = \frac{8I_2}{5}$ hence $y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = \frac{-0.8I_2}{1.6I_2} \Big|_{V_1=0} = -0.5S$

Note that each of these parameters is the ratio of a current to a voltage and therefore is an admittance with the dimension of siemens; this is why they are called y -parameters.

Self Test 42:

- a) Determine the z-parameters for the circuit in Fig. 42-7
- b) Determine the y-parameters for the circuit in Fig. 42-8

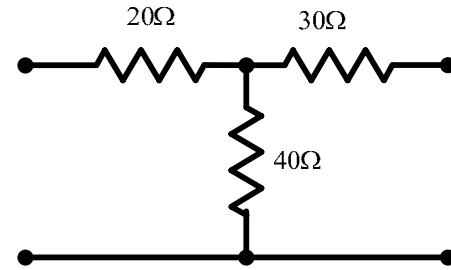


Fig. 42-7 Circuit for self test 42a

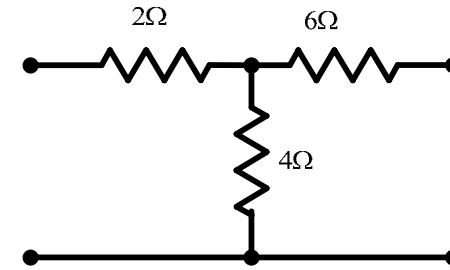


Fig. 42-8 Circuit for self test 42b

Answer:

- a) $z_{11} = 60\Omega$ $z_{12} = 40\Omega$ $z_{22} = 70\Omega$ $z_{21} = 40\Omega$
- b) $y_{11} = 0.2273S$ $y_{12} = y_{21} = -0.0909S$ $y_{22} = 0.1364S$