10-1. Find the ratio of the velocities of two vehicles, one powered by a liquid-chemical rocket and the other by a solid-chemical one, when they are used for acceleration of a 10,000-kg payload in a zero-gravity field. Both vehicles have a total initial mass of 510,000 kg. The liquid-propellant rocket has 60% greater specific impulse and 30% greater mass of empty vehicle (without propellant and payload), and the solid-propellant rocket has a structural coefficient of $\epsilon = 0.080$.

$$\Delta u = u_e \ln\left(\frac{m_o}{m_s + m_L}\right) = g_e I_{sp} \ln\left(\frac{m_o}{m_s + m_L}\right)$$

$$\frac{(I_{sp})_{liq}}{(I_{sp})_{solid}} = 1.6 \qquad m_o = 510{,}000 \text{ kg} \qquad m_L = 10{,}000 \text{ kg}$$

$$\epsilon_{solid} = 0.08 = \frac{(m_s)_{solid}}{m_s + m_p} = \frac{(m_s)_{solid}}{m_o - m_L}$$

$$\therefore (m_s)_{solid} = 0.08(500{,}000) = 40{,}00$$

$$(m_s)_{liquid} = 1.3\,(40{,}000) = 52{,}0c$$

$$R_{solid} = \frac{m_o}{(m_s)_{solid} + m_L} = \frac{510{,}000}{40{,}000 + 10{,}000} = 10.20$$

$$R_{liq} = \frac{m_o}{(m_s)_{liq} + m_L} = \frac{510{,}000}{52{,}000 + 10{,}000} = 8.221$$

$$\frac{(\Delta u)_{liq}}{(\Delta u)_{solid}} = \frac{(I_{sp})_{liq}}{(I_{sp})_{solid}} \frac{\ln R_{liq}}{\ln R_{solid}} = 1.6 \frac{\ln 8.22}{\ln 10.2} = \underline{\underline{1.45}}$$

10-2. A sounding rocket is to be launched vertically from the earth's surface. It is to be designed for a 350-kg payload that must not suffer an acceleration greater than $5g$ during the burning period. The maximum propellant mass is 1000 kg, and the structural efficiency factor ϵ is 0.1. The solid propellant rocket motor that will be designed for this vehicle is expected to have a specific impulse of 250 s.

Taking $g = g_e$ and neglecting aerodynamic drag, determine:

a. The minimum allowable burning period,
b. The maximum height attainable,
c. The height allowable if the maximum payload acceleration were limited to $4g$

$$\text{Eq (10-15)} \qquad u = -u_e \ln\left[1 - \left(1 - \frac{1}{R}\right)\frac{t}{t_b}\right] - g_e t$$

$$a = \frac{du}{dt} = u_e \frac{\left(1 - \frac{1}{R}\right)\frac{1}{t_b}}{1 - \left(1 - \frac{1}{R}\right)\frac{t}{t_b}} - g_e$$

a is maximum for $t = t_b$

$$a_{max} = u_e \frac{\left(1 - \frac{1}{R}\right)\frac{1}{t_b}}{1 - \left(1 - \frac{1}{R}\right)} - g_e = u_e \frac{(R-1)}{t_b} - g_e$$

$$\text{Min allowable} \quad t_b = \frac{u_e(R-1)}{a_{max} + g_e} = \frac{I_{sp}(R-1)}{a_{max}/g_e + 1}$$

With $m_p = 1000$, $m_L \doteq 350\,kg$ and $\epsilon = 0.1$ $m_s = 111\ kg$

$$\epsilon = 0.1 = \frac{m_s}{m_s + m_p} = \frac{m_s}{m_s + 1000}$$

$$R = \frac{m_o}{m_s + m_L} = \frac{1000 + 111 + 350}{111 + 350} = 3.17$$

For $\frac{a_{max}}{g_e} = 5$ $\quad t_b = \frac{250(3.17 - 1)}{1 + 5} = 90.4\,s$

(For $\frac{a_{max}}{g_e} = 4$ $\quad t_b = 108.5\,s$)

Eq. (10-18)

$$h_{max} = \frac{u_e^2 (\ln R)^2}{2 g_e} - u_e t_b \left(\frac{R}{R-1} \ln R - 1 \right)$$

$$= \frac{u_e^2 (\ln R)^2}{2 g_e} - \frac{u_e^2 (R-1)}{a_{max} + g_e} \left(\frac{R}{R-1} \ln R - 1 \right)$$

$$= g_e I_{sp}^2 \left[\frac{(\ln R)^2}{2} - \frac{R \ln R - R + 1}{\frac{a_{max}}{g_e} + 1} \right]$$

For $\frac{a_{max}}{g_e} = 5$

$$h_{max} = 9.81 (250)^2 \left[\frac{(\ln 3.17)^2}{2} - \frac{3.17 \ln 3.17 - 3.17 + 1}{6} \right]$$

$$= \underline{\underline{256}}\ km$$

For $\frac{a_{max}}{g_e} = 4$ $\quad h_{max} = \underline{\underline{226}}\ km$

10-4. Show that if engine mass were a negligible part of the entire total mass of a rocket vehicle, and if the total structural and engine mass were simply proportional to the initial propellant mass, the structural efficiency factor ϵ would be independent of the mission Δu. A single-stage solid-propellant booster has the following characteristics:

$$\frac{\mathcal{M}_{engine}}{\mathcal{M}_0} = 0.005\,,$$

$$\frac{\mathcal{M}_{tank}}{\mathcal{M}_p} = 0.03\,,$$

$$I_{sp} = 200\,.$$

Show how the payload ratio λ and the structural efficiency ϵ depend on the mission velocity Δu and compare your results for ϵ with Fig. 10.6. The definition of the structural factor in this case may be written

$$\epsilon = \frac{\mathcal{M}_{tank} + \mathcal{M}_{engine}}{\mathcal{M}_{propellant} + \mathcal{M}_{tank} + \mathcal{M}_{engine}}.$$

Here neglect gravity and drag effects.

$$\epsilon = \frac{m_{tank} + m_{engine}}{m_p + m_{tank} + m_{engine}}$$

(a) Suppose $m_s = m_{tank} + m_{engine} = k\, m_p$

then $\epsilon = \frac{k}{1+k}$ (independent of Δu)

(b) $\frac{m_{engine}}{m_o} = 0.005$, $\frac{m_{tank}}{m_p} = 0.03$, $I_{sp} = 200$

then
$$\epsilon = \frac{\left(\frac{m_{tank}}{m_p}\right)\frac{m_p}{m_o} + \frac{m_{engine}}{m_o}}{\left(1 + \frac{m_{tank}}{m_p}\right)\frac{m_p}{m_o} + \frac{m_{engine}}{m_o}}$$

also
$$\Delta u = u_e \ln \frac{m_o}{m_o - m_p}$$

so
$$\frac{m_p}{m_o} = 1 - e^{-\Delta u/u_e} \qquad (1)$$

and
$$\epsilon = \frac{\left(\frac{m_{tank}}{m_p}\right)\left(1 - e^{-\Delta u/u_e}\right) + \frac{m_{engine}}{m_o}}{\left(1+\frac{m_{tank}}{m_p}\right)\left(1 - e^{-\Delta u/u_e}\right) + \frac{m_{engine}}{m_o}} \qquad (2)$$

$$\lambda = \frac{m_L}{m_o - m_L} = \frac{m_o - m_{tank} - m_{engine} - m_p}{m_{tank} + m_{engine} + m_p}$$

$$= \frac{1}{\left(1 + \frac{m_{tank}}{m_p}\right)\frac{m_p}{m_o} + \frac{m_{engine}}{m_o}} - 1$$

$$\lambda = \frac{1}{\left(1 + \frac{m_{tank}}{m_p}\right)\left(1 - e^{-\Delta u/u_e}\right) + \frac{m_{engine}}{m_o}} - 1 \qquad ③$$

Equation	①	②	③
Δu	$\frac{m_p}{m_o}$	ϵ	λ
2000	0.639	.0364	0.508
4000	0.870	.0345	0.110
6000	0.953	.0340	0.013
8000	0.983	.0339	(-ve)

10-5. A vehicle that utilizes a liquid-chemical rocket is to be used for escape of a payload of 100,000 kg from the earth. The vehicle is to have two stages, each with $\epsilon = 0.06$ and $I_{sp} = 400$ s, and both having the same structural coefficient λ. Estimate the total mass of the vehicle. Assume as a first approximation that $g \cos \theta t_b = 2000$ m/s and that drag effects are negligible. To ensure acceptable launch acceleration (say $0.2 g_e$), what restriction must be placed on the burning period for the first stage?

with $\lambda_1 = \lambda_2 = \lambda$ and $\epsilon_1 = \epsilon_2 = \epsilon = 0.06$

$$\Delta u = 2 u_e \ln\left(\frac{1+\lambda}{\epsilon+\lambda}\right)$$

$$\Delta u = \sqrt{\frac{2GM}{r_e}} + \overline{g \cos\theta}\, t_b$$

$$= \sqrt{2 r_e g_e} + \overline{g \cos\theta}\, t_b$$

$$= \sqrt{12{,}742{,}460(9.81)} + 2000 \qquad \left(\begin{matrix} r_e \text{ from} \\ \text{Table 10.7}\end{matrix}\right)$$

$$= 13{,}180 \text{ m/s}$$

$$\frac{1+\lambda}{\epsilon+\lambda} = \exp\left\{\frac{\Delta u}{2u_e}\right\} = \exp\left\{\frac{13{,}180}{2(9.81)400}\right\} = 5.362$$

$$\frac{1+\lambda}{0.06+\lambda} = 5.362 \qquad \text{so} \quad \lambda = 0.1555$$

$$\lambda = \frac{m_L}{m_o - m_L}$$

$$\lambda = \frac{m_L}{m_{02} - m_L} = \frac{m_{02}}{m_{01} - m_{02}} = 0.1555$$

So $\quad m_{02} = m_L \left(\frac{1+\lambda}{\lambda}\right), \quad m_{01} = m_{02}\left(\frac{1+\lambda}{\lambda}\right) = m_L \left(\frac{1+\lambda}{\lambda}\right)^2$

$$m_{01} = 100{,}000 \left(\frac{1.1555}{0.1555}\right)^2 = 5{,}522{,}000 \text{ kg}$$

Required thrust at lift-off

$$\mathbb{T} = m_{01}(a + g_e) = \dot{m} u_e$$

With uniform burning rate (1st stage)

$$m_{01}(a + g_e) = \frac{m_{01} - m_{02}}{t_b} u_e$$

$$t_b = \frac{\left(1 - \frac{m_{02}}{m_{01}}\right) u_e}{a + g_e} = \frac{\left(1 - \frac{\lambda}{1+\lambda}\right) u_e}{a + g_e} = \frac{u_e}{(1+\lambda)(a + g_e)}$$

for $a = 0.2 g_e$

$$t_b = \frac{400}{1.1555(0.2 + 1)} = \underline{\underline{288}}\ \text{s}$$